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Study on the Application of the Combined Prediction Modeling Method to Thermal Error Modeling on NC Machine Tools

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Abstract. Due to the complexity of machine tool thermal errors affected by various factors, a new combining prediction model, based on the theory of gery system GM (1,1) model, is applied to the trend prediction of machine tool thermal errors. The degree of smoothness of primary data sequence is first improved by function transform method and sequentially grey system GM (1,1) model is established; second, time series analysis model is established by remnant sequence of GM (1,1) model to amend the precision of grey system GM (1,1) model. Thus, the precision of combining prediction model is further improved. Through the prediction study on thermal error modeling in a spot NC turning center, testing results showed that combining prediction model can highly improve machine tool's prediction precision and make it more effective for real-time compensation of machine tool thermal error.

Introduction

With the rapid development of modern mechanical manufacturing technique, the processing precisions of all types of NC machine tools and processing centers are more highly required. The problem of machine tool errors caused by thermal distortion becomes more and more significant. A large number of researches show that thermal error is the largest source of machining error and amounts to 40%-70% [1]. Hence, how to control the thermal errors of machine tool becomes essential in order to increase their processing precision. There are two methods for improving precision of machine tool: error prevention and error compensation. That is, error prevention is to eliminate potential error source through advanced design and manufacture technique to realize aim of precision, but its cost is higher and its effect is not ideal; error compensation mainly amends error through the measurement and indirect kinematics model of machine tool' error without improving machine tool' self precision. Therefore, error compensation technique, as an effective and economical means, has been one of interests of related studies all over the world [2-4].

In the technology of thermal error compensation on machine tools, machine tool is a complicated system of thermal state and is affected by various factors such as processing condition and cycle, the use of coolant as well as the environment. Moreover, due to the non-linearity and interaction of machine tool thermal error, it is difficult to use a individual prediction method to accurately describe thermal errors' moving trend [5-6]. A new combining prediction model [7], based on the theory of gery system GM (1,1) model [8], is applied to the trend prediction modeling of machine tool thermal errors. The degree of smoothness of primary data sequence is first improved by function transform method and sequentially grey system GM (1,1) model is established; second, time series analysis model [9] is established by remnant sequence of GM (1,1) model to amend the precision of grey system GM (1,1) model. Thus, the precision of combining prediction model is further improved. Through the prediction study on thermal error modeling in a spot NC turning center, testing results showed that combining prediction model can highly improve machine tool's prediction precision and make it more effective for real-time compensation of machine tool thermal error.

The establishment of prediction model of machine tool thermal error

Grey System Theory Modeling. Assuming $\{x^{(0)}(i)\}$ $(i=1,2,\cdots,n)$ is a primary data sequence, after once accumulating generation, it becomes a new sequence as follows:

$$x^{(1)}(k)=\sum_{j=1}^{k}x^{(0)}(j) \qquad k=1,\ 2,\ \cdots,\ n \tag{1}$$

Then corresponding differential equation of GM (1,1) model is expressed as:

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=u \tag{2}$$

where a is development grey number, u is endogenetic control grey number.

$\hat{a}$ is defined as estimating parameter vector, $\hat{a}=\begin{bmatrix} a \\ u \end{bmatrix}$, according to least square method, using following formula:

$$\hat{a}=\left(B^{T}B\right)^{-1}B^{T}Y_{n} \tag{3}$$

where $B=\begin{bmatrix} -\frac{1}{2}\left[x^{(1)}(1)+x^{(1)}(2)\right] & 1 \\ -\frac{1}{2}\left[x^{(1)}(2)+x^{(1)}(3)\right] & 1 \\ \cdots & \cdots \\ -\frac{1}{2}\left[x^{(1)}(n-1)+x^{(1)}(n)\right] & 1 \end{bmatrix}$, $Y_n=\left(x^{(0)}(2),x^{(0)}(3),\cdots,x^{(0)}(n)\right)^{T}$

to resolve differential equation (2), the solution is expressed as

$$\hat{x}(k+1)=\left[X^{(0)}(1)-\frac{u}{a}\right]e^{-ak}+\frac{u}{a} \qquad k=0,1,2,\cdots,n \tag{4}$$

equation (4) is just GM (1,1) model and it is carried on once inverse accumulating generation, the reductive model is got by

$$\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k) \quad \text{or} \quad \hat{x}^{(0)}(k+1)=\left[x^{(0)}(1)-\frac{u}{a}\right]e^{-ak}\left(1-e^{a}\right) \tag{5}$$

Function transform modeling of grey system. Due to the fact that grey system model requires the degree of smoothness of disperse data sequence, for non-minus primary data $\{x^{(0)}(i)\}$ $(i=1,\ 2,\ \cdots,\ n)$, $f(i)$, as a non-minus and monotonously degressive sequence, would be chosen to transform $x^{(0)}(i)$, a new sequence is given by:

$$F\left(x^{(0)}(i)\right)=x^{(0)}(i)*f(i) \tag{6}$$

which would evidently improve the degree of smoothness of primary data $x^{(0)}(i)$; then $F\left(x^{(0)}(i)\right)$ would be modeled by grey system model and be sequentially reverted by contrary transform, finally, modeling precision of system would be improved significantly [7].

Time Series Analysis Modeling. Assuming $\{x_t\}=\{x_1,x_2,\cdots,x_n\}$ is a primary data sequence, after the operation of smoothness and zero-equal-value, a linear and random difference equation could be set up as follows:

$$x_t-\sum_{i=1}^{p}\phi_i x_{t-i}=a_t-\sum_{j=1}^{q}\theta_j a_{t-j}: \qquad a_t-\text{NID}\ (0,\ \sigma_a^2) \tag{7}$$

where ϕ_i ($i=1,2,\cdots,p$) is self-regress parameter, p is self-regress rank number; θ_j ($j=1,2,\cdots,q$) is gliding average parameter, q is gliding average rank number; a_t is remnant or random disturbance, that is, a zero-equal-value, normal independent distribution random sequence ; σ_a^2 is normal distribution square difference.

The equation (7) is just called p rank self-regress and q rank slide average combined model, namely $ARMA(p,q)$ model, which could be transformed into two special cases: $MA(q)$ model and $AR(p)$ model.

Combining prediction model. The establishment of combining prediction model is generalized as: first, primary data sequence is pretreated through function transform method to improve its degree of smoothness; second, new data sequence obtained is used to establish grey system GM (1,1) model, and the prediction formula of primary data sequence $\{x_i^{(0)}\}(i=1, 2, \cdots, n)$ is expressed as:

$$d_i = \left[x^{(0)}(1) - \frac{u}{a}\right]e^{-ai}\left(1-e^{a}\right) \tag{8}$$

third, remnant sequence $\{\varepsilon_i\}$ is used to establish time series analysis model, due to the fact that $AR(p)$ model could be set up easily and has less calculating quantity and it's parameters could be estimated more exactly [10], $AR(p)$ model is applied to predict remnant sequence $\{\varepsilon_i\}$ and corresponding prediction formula is showed as:

$$x_t(l) = \phi_1 x_t(l-1) + \phi_2 x_t(l-2) + \cdots\cdots + \phi_p x_t(l-p) \tag{9}$$

where l is predicting step number, $l = 1,2,3,\cdots$

therefore, d_i and ε_i make up of combining prediction model as follows:

$$x_i = d_i + \varepsilon_i \tag{10}$$

prediction formula of combining prediction model is expressed as:

$$\hat{x}_i(l) = \left[x^{(0)}(1) - \frac{u}{a}\right]e^{-a(i+l-1)}\left(1-e^{a}\right) + \hat{\varepsilon}_i(l) \tag{11}$$

where $\hat{x}_i(l)$ is the predicted value of step l, at the time i.

Application and Analysis of Combining Prediction Model

Measurement and collection of Machine Tool Thermal Errors. The method is applied to a

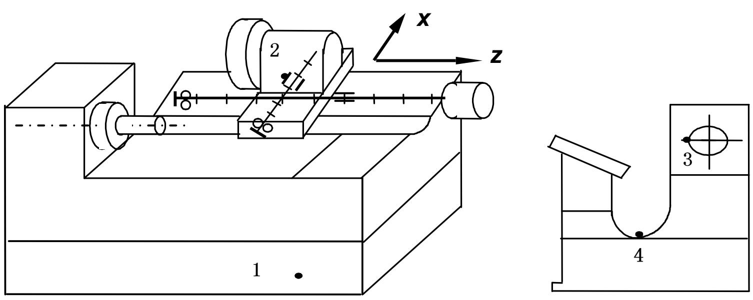

Fig. 1 Sketch map of turning center

NC turning center, as shown in Fig. 1, and is used in the study of thermal error compensation. Sketch map of thermal error's measurement is shown in Fig. 2.

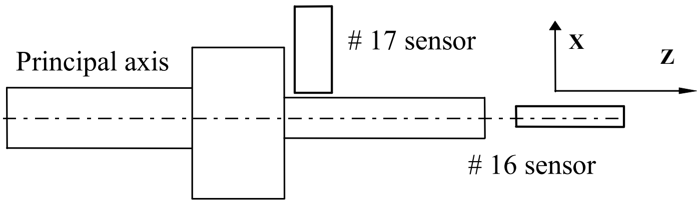

Fig. 2 Measurement of thermal errors of principal axis

Two displacement sensors fixed on knife rest measure thermal errors between principal axis of X direction (17) and Z direction (16) and knife rest. For workpiece is very short, the error of leaning angle is neglected.

Prediction Analysis of Combining Prediction Model. The text chooses radial thermal errors in the process of turning as researchful objects (for workpieces' precision of radial dimension has rather higher request, the radial thermal error of machine tool is only considered), in practice, researchful objects are workpieces' errors of radial dimension changed along with the machine tool's temperature or the time. Change figure of thermal errors of turning center is shown in Fig. 3.

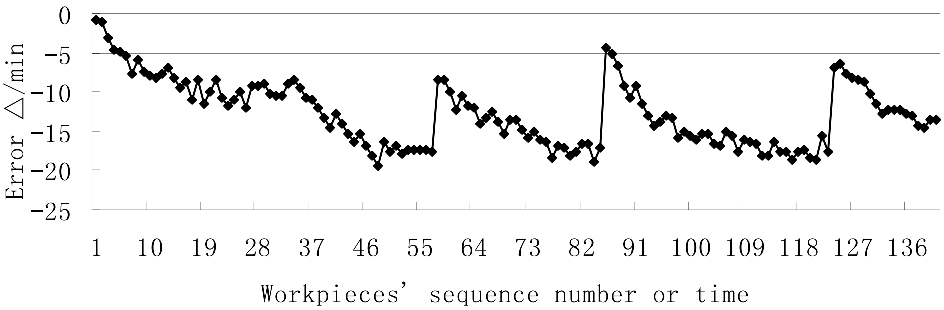

Fig. 3 Change figure of thermal errors of turning center

Based on the above mentioned combining prediction model, for data sequence shown in Fig. 3, the former 100 data of data sequence are used to establish combining prediction model according to equation (1) - equation (11); then, combining prediction model predicts the latter 41 data of primary data sequence, predicting value of machine tool thermal error would be obtained. In order to illuminate well prediction precision of combining prediction model for machine tool thermal error, time series analysis $AR(p)$ model and grey system theory GM (1,1) model are also used to model and predict thermal error. The comparisons of predicted value and practical value of all models are drawn in Fig. 4-Fig. 6 according to above predicted results. In addition, the peak values of both practical values of thermal errors and predicted values obtained from three types of models are listed in Table. 1.

Table. 1 The peak values of machine tool thermal errors

Appraisal target	Primary error	Prediction errors		
		Gery system theory	Time series analysis	Combining prediction model
Peak value (μm)	-18.6	-11.86	-9.92	-8.13

It can be seen from 3 figures and Table. 1 clearly, predicting precision of combining prediction model is highest. Thermal error of turning center can be kept in -8.13 μm from primary –18.6 μm through error compensation using combining prediction model. Predicting precision of combining prediction model is 18% and 31.4% higher than those of time series analysis model and grey system theory GM (1,1) model respectively. The comparison of error curve of three grey prediction models is shown in Fig. 7.

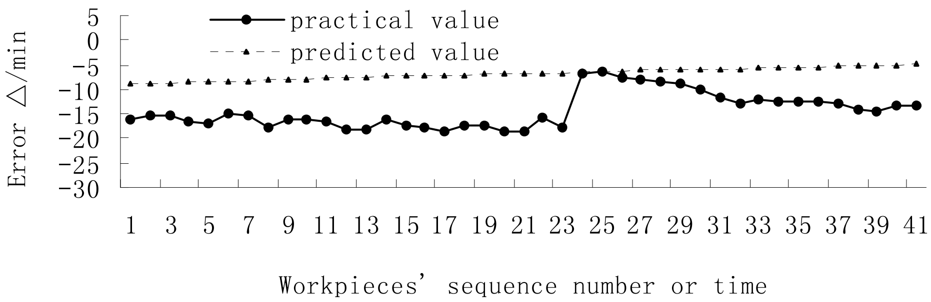

Fig. 4 Comparison of predicted curve and practical curve of grey system theory model

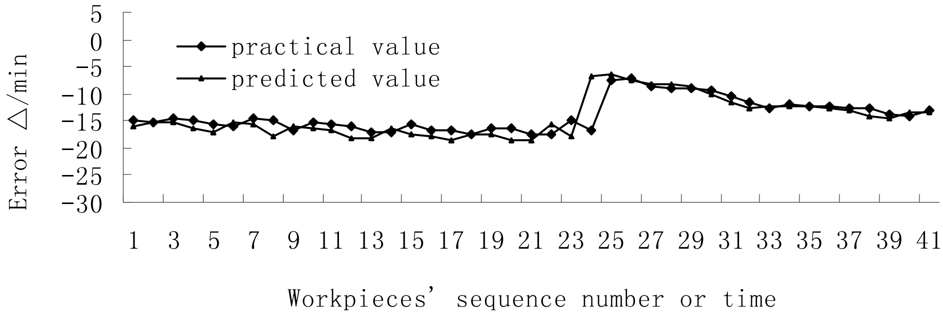

Fig. 5 Comparison of predicted curve and practical curve of time series analysis model

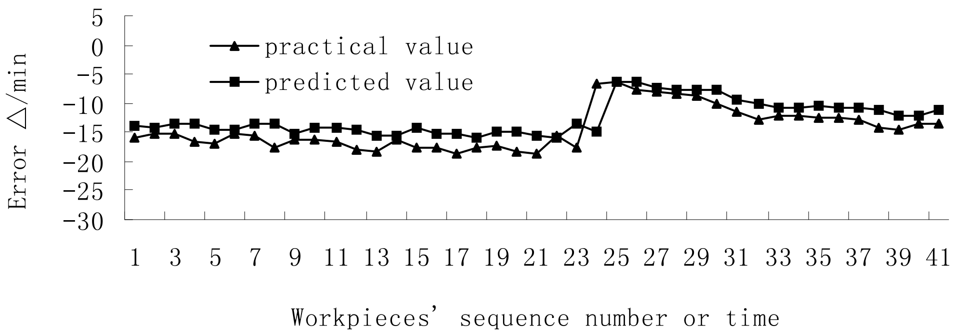

Fig. 6 Comparison of predicted curve and practical curve of combining prediction model

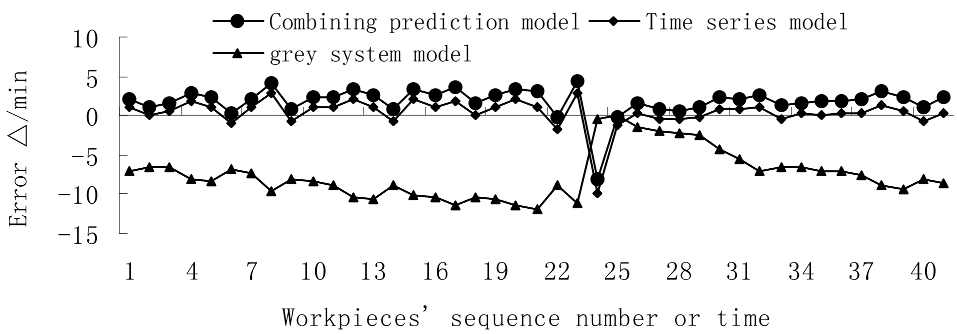

Fig. 7 Comparison of errors curve of three kinds of prediction model

Summary

(a) Combining prediction model, making full use of advantages of both time series analysis and grey system theory, reflects not only random factors' influence of machine tool thermal errors but also change tendency of machine tool thermal errors. The testing results show that prediction precision of combining prediction model is higher than either of two prediction methods, so it highly improves machine tool's processing precision and makes it more effective for real-time compensation of NC thermal error.

(b) Combining prediction model improves the degree of smoothness of primary data sequence through function transform method. Sequentially, the precision of prediction model is also improved. If a more precise transform function is available to improve the degree of smoothness, the precision of prediction model could be further improved.

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