

Low complex direction of arrival estimation method based on adaptive filtering algorithm

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Abstract: Conventional subspace decomposition-based direction of arrival (DOA) estimation methods require eigenvalue decomposition of spatial covariance matrix, therefore these methods are computationally intensive, and their implementation is difficult in real-time applications. However, a DOA estimation method based on fixed step size least mean square (LMS) algorithm has overcome this deficiency but the suitable selection of step size is very difficult. A DOA estimation method based on the variable step size LMS algorithm is proposed in this article. In the proposed method, the step size is updated by using the estimated error signal. The spatial spectrum is obtained by the reciprocal of array pattern, where the peak values indicate the estimated DOAs of signals. The proposed method can provide better performance with low computational cost. The performance of proposed method is verified by the simulations and compared with some existing methods.

1 Introduction

Direction of arrival (DOA) estimation has wide application in sonar, radar, communication and navigation systems. The conventional subspace decomposition-based DOA estimation methods, i.e. multiple signal classification (MUSIC) [1] and estimation of signal parameters via rotational invariance techniques [2] are very popular due to their better performance. However, these methods require eigenvalue decomposition (EVD) of spatial covariance matrix, which is a computationally intensive procedure. Moreover, these methods also require to estimate the number of sources [3]. Therefore, various methods were proposed in [4–6] to minimise the computational complexity of conventional methods. However, these methods reduce the computational complexity to some extent, but EVD of approximated covariance matrix is still required. To overcome the complexity problem of subspace decomposition-based methods, a DOA estimation method based on fixed step size least mean square (FSS-LMS) algorithm was proposed in [7]. This method is computationally efficient but the suitable value of fixed step size is very difficult to choose.

Therefore, a DOA estimation method based on the variable step size LMS (VSS-LMS) is proposed in this paper. The step size is updated by using the estimated error signal. The spatial spectrum is obtained by the reciprocal of array pattern, where the peak values indicate the estimated DOAs of the signals. The proposed method provides better performance with low computational complexity, because it has no need of EVD and the estimation of number of sources.

Rest of the paper is organised as signal model is described in Section 2. The methodology of proposed method is explained in Section 3. The simulation results are discussed in Section 4. Finally, the paper is concluded in Section 5.

2 Signal model

A uniform linear array (ULA) of N omnidirectional antenna elements having half wavelength spacing is considered, where $M (M < N)$ far-field uncorrelated narrowband signals with K number of snapshots from the directions $\theta = [\theta_0, \theta_2, \dots, \theta_{M-1}]$. The first element of array is used as a reference element, whereas remaining $N - 1$, elements are considered as the auxiliary array

elements as shown in Fig. 1. The received $\mathbf{x}(k) \in \mathbb{C}^{N \times 1}$ can be written as

$$\mathbf{x}(k) = \sum_{m=0}^{M-1} s_m(k) \mathbf{a}(\theta_m) + \mathbf{v}(k), \quad (1)$$

where $s_m(k)$ are the corresponding signal envelopes, $m = 0, 1, 2, \dots, M - 1$, $\mathbf{a}(\theta_m)$ is the array steering vector of the form, $\mathbf{a}(\theta_m) = [1, e^{j\pi \sin \theta_1}, \dots, e^{j\pi(N-1) \sin \theta_{M-1}}]$ and $\mathbf{v}(k)$ is zero mean additive white Gaussian noise. The matrix form of (1) can be expressed as

$$\mathbf{x}(k) = \mathbf{A} \mathbf{s}_m(k) + \mathbf{v}(k), \quad (2)$$

where \mathbf{A} is an array manifold matrix. The signal received at reference element $x_0(k)$ and the signals received at auxiliary elements $\hat{\mathbf{x}}(k)$ can be written as

$$x_0(k) = s_0(k) + \sum_{m=1}^{M-1} s_m(k) + v_0(k). \quad (3)$$

$$\hat{\mathbf{x}}(k) = s_0(k) \mathbf{a}(\theta_0) + \sum_{m=1}^{M-1} s_m(k) \mathbf{a}(\theta_m) + \mathbf{v}(k). \quad (4)$$

3 Methodology

In this section, the proposed DOA estimation method based on VSS-LMS algorithm is derived, where the weight vectors for auxiliary elements are calculated recursively and expressed by

$$\hat{\mathbf{w}}(k+1) = \hat{\mathbf{w}}(k) + \mu_{\text{vss}}(k) \hat{\mathbf{x}}(k) e^*(k), \quad (5)$$

where $\mu_{\text{vss}}(k)$ is the proposed variable step size, which is given by

$$\mu_{\text{vss}}(k) = \left[1 - \frac{1}{\exp(|e(k)|)} \right] [\delta_e^2(k) + \delta_x^2(k)]^{-1}, \quad (6)$$

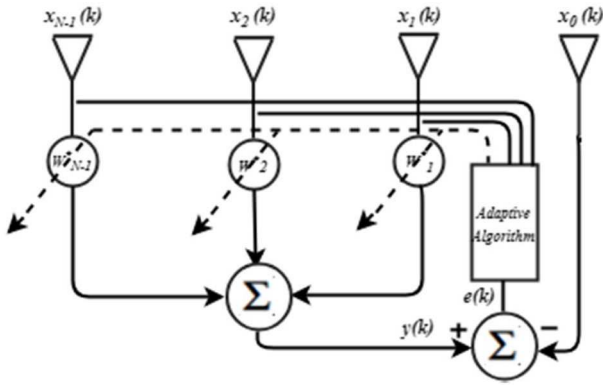


Fig. 1 Schematic diagram of adaptive antenna array

Table 1 Summary of proposed method

inputs:	$x(k)$ received signal $x_0(k)$ reference signal $\hat{x}(k)$ auxiliary array signal
initialisation:	$\hat{w}(0) = 0, \delta_e^2(0) = 0$
output:	$y(k) = \hat{w}^H(k)\hat{x}(k)$ $e(k) = x_0(k) - y(k)$ $\delta_e^2(k+1) = \delta_e^2(k) + e^2(k) $ $\delta_{\hat{x}}^2(k) = \hat{x}^H(k)\hat{x}(k)$ $\mu_{vss}(k) = \left[1 - \frac{1}{\exp(e(k))}\right] [\delta_e^2(k) + \delta_{\hat{x}}^2(k)]^{-1}$ $\hat{w}(k+1) = \hat{w}(k) + \mu_{vss}(k)\hat{x}(k)e^*(k)$
array pattern:	$G(\theta) = w^H a(\theta) , w = [1; \hat{w}]$
spatial spectrum:	$P_{vss}(\theta) = \frac{1}{ w^H a(\theta) }$

Table 2 Computational complexities of DOA estimation methods

Algorithms	Multiplications
MUSIC	$O(MN^2 + N(N-M)L + (N-M)L)$
FSS-LMS	$O(K(2N-1) + NL)$
proposed	$O(K(3N) + NL)$

where 'exp' is an exponential function, $e(k)$ is estimated error signal, cumulative sum of squared error $\delta_e^2(k)$, and $\delta_{\hat{x}}^2(k)$ is estimated signal power, which are calculated by

$$e(k) = x_0(k) - \hat{w}^H(k)\hat{x}(k), \quad (7)$$

$$\delta_e^2(k+1) = \delta_e^2(k) + |e^2(k)|, \quad (8)$$

$$\delta_{\hat{x}}^2(k) = \hat{x}^H(k)\hat{x}(k), \quad (9)$$

where $(\cdot)^H$ is conjugate transpose. The proposed method can choose step size adaptively according to (6), it gets large step size at early stage that accelerates the convergence speed, when it converges to optimal solution, it achieves smaller step size that ensures the stability and low mean square error. After obtaining the optimal weight vector, the array pattern is calculated by

$$G(\theta) = w^H a(\theta), \quad (10)$$

where $w \in \mathbb{C}^{N \times 1} = [1; \hat{w}]$ is whole array weight vector. Finally, the spatial spectrum [7] is obtained by the reciprocal of (10), which is given by

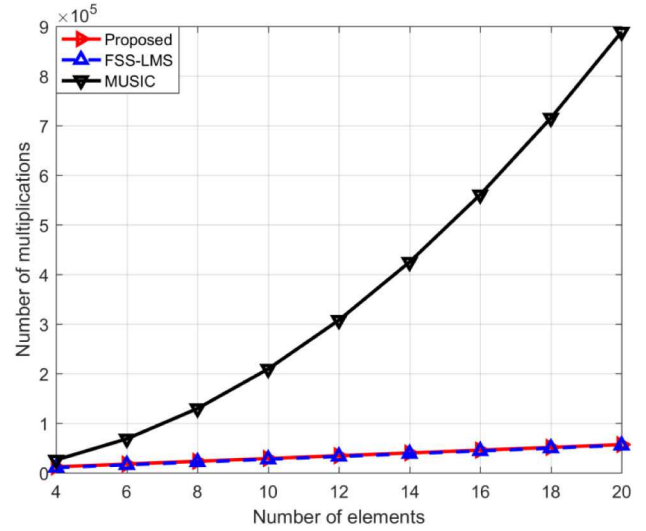


Fig. 2 Comparison of the computational complexity of the proposed, FSS-LMS and MUSIC methods versus number of elements with 500 snapshots

$$P_{vss}(\theta) = \frac{1}{|w^H a(\theta)|}. \quad (11)$$

The proposed method is summarised in Table 1.

4 Simulation results

In this section, the performance of proposed method is verified by the numerical simulations. In these simulations, a ULA with eight elements having half-wavelength spacing is considered. Two uncorrelated signals with signal-to-noise ratio (SNR) 10 and 20 dB are received from 30° and 60° , respectively. The step size for the FSS-LMS method is 2×10^{-5} . The spatial spectrum is obtained from one Monte Carlo simulation, whereas the root mean square error (RMSE) is calculated from an average of 200 Monte Carlo simulations.

The computational complexity of proposed method is investigated and compared with FSS-LMS and MUSIC methods, whereas summarised in Table 2 and plotted in Fig. 2, where L is number of angles to be scanned. It is found that the computational complexity of proposed method is much less than MUSIC method and almost same as FSS-LMS method.

The spatial spectrum of proposed method is investigated and compared with FSS-LMS and MUSIC methods, and corresponding results are shown in Figs. 3 and 4. It is found that the proposed method can estimate the DOAs of the signals accurately and provide narrow peaks similar to MUSIC method, whereas FSS-LMS method cannot estimate the DOAs of signals accurately and gives error of 0.2° at 30° and 60° .

The RMSE is calculated by

$$\text{RMSE}(\theta) = \sqrt{\frac{1}{TQ} \sum_{t=1}^T \sum_{m=1}^M (\hat{\theta}_m(t) - \theta_m(t))^2}, \quad (12)$$

where $\theta_m(t)$ represents the real DOAs, $\hat{\theta}_m(t)$ is the estimated value of DOAs for the t th trail, $t = 1, 2, \dots, T$ and T represents the independent Monte Carlo simulations. The RMSE of proposed method versus SNR is investigated and compared with FSS-LMS and MUSIC methods, and corresponding results are shown in Fig. 5. It is found that the RMSE of proposed method is decreasing rapidly with increasing SNR similar to MUSIC, however the proposed method outperforms the FSS-LMS method and its performance is close to MUSIC method. The RMSE of proposed method versus number of elements is investigated and compared with FSS-LMS and MUSIC methods, and corresponding results are shown in Fig. 6. It is found that the performance of proposed method is much better than FSS-LMS method and close to MUSIC method. The RMSE of proposed method versus number of

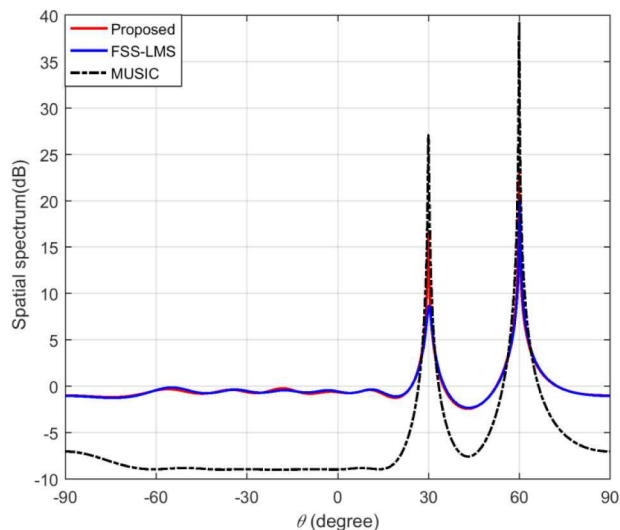


Fig. 3 Spatial spectrum of the proposed, FSS-LMS and MUSIC methods with 8 elements, 500 snapshots and two signals with the SNR of 10 and 20 dB from 30° and 60°

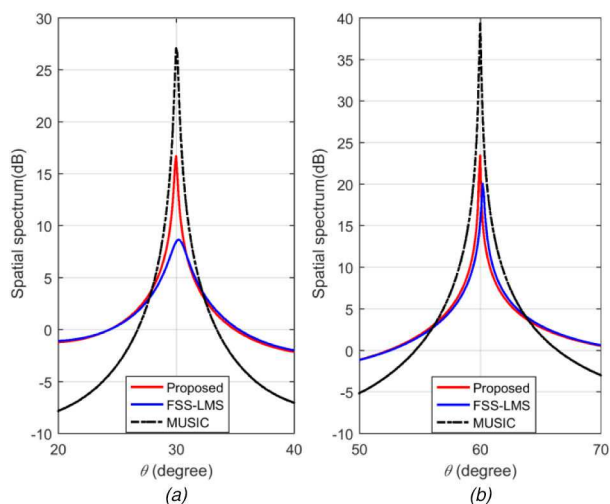


Fig. 4 Spatial spectrum of the proposed, FSS-LMS and MUSIC methods with two signals of 10dB and 20dB, 8 elements, and 500 snapshots
(a) Enlarged spatial spectrum of the proposed, FSS-LMS and MUSIC methods ~30°
(b) Enlarged spatial spectrum of the proposed, FSS-LMS and MUSIC methods ~60°

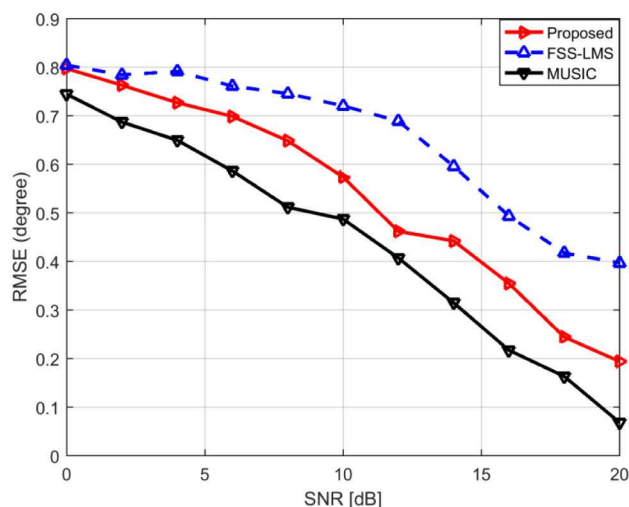


Fig. 5 RMSE of the proposed, FSS-LMS and MUSIC methods versus SNR with 8 elements, 500 snapshots and two signals from 30° and 60°

snapshots is investigated and compared with FSS-LMS and MUSIC methods, and corresponding results are shown in Fig. 7. It

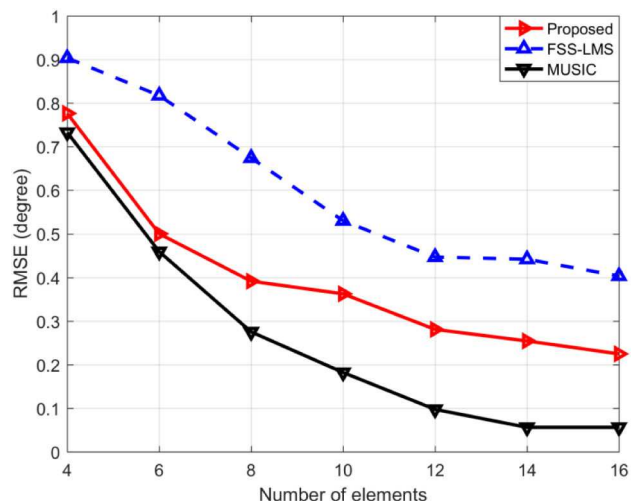


Fig. 6 RMSE of the proposed, FSS-LMS and MUSIC methods versus number of elements with 500 snapshots and two signals with the SNR of 15 dB from 30° and 60°

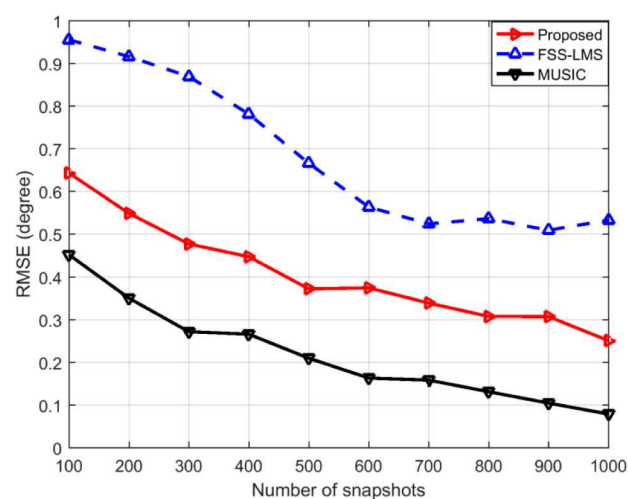


Fig. 7 RMSE of the proposed, FSS-LMS and MUSIC methods versus number of snapshots with 8 elements and two signals with the SNR of 15 dB from 30° and 60°

is found that the performance of proposed method is also much better than FSS-LMS method, but the performance of MUSIC method is slightly better than proposed method, however the computational complexity of proposed method is very less than MUSIC method.

5 Conclusion

A DOA estimation method based on VSS-LMS algorithm has been proposed in this paper. In the proposed method, the step size is derived by using the estimated error signal. The spatial spectrum is obtained by the reciprocal of the array pattern, where the peaks values indicate the estimated DOAs of the signals. The performance of proposed method has been verified by the simulations. It has been found that the proposed method can provide better performance with low computational complexity.

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7 References

- [1] Schmidt, R. O.: 'Multiple emitter location and signal parameter estimation', *IEEE Trans. Antennas Propag.*, 1986, **34**, (3), pp. 276–280

- [2] Roy, R., Kailath, T.: 'Esprit-estimation of signal parameters via rotational invariance techniques', *IEEE Trans. Acoust., Speech, Signal Process.*, 1989, **37**, (7), pp. 984–995
- [3] Li, S, Yang, X, Li, W, *et al.*: 'Estimation of source number based on power inversion and adaptive threshold in colored noise', 2016 CIE Int. Conf. on Radar (RADAR), Guangzhou, 2016, pp. 1–5
- [4] Qian, C., Huang, L.: 'A low-complexity nystrom-based algorithm for array subspace estimation', Proc. 2nd Int. Conf. on Instrumentation, Measurement, Computer, Communication and Control (IMCCC), Harbin, China, December 2012, pp. 112–114
- [5] Guo, Y., Li, W., Zuo, Y., *et al.*: 'A nystrom-based esprit algorithm for DOA estimation of coherent signals'. Proc. Int. Sym. Antennas Propagation, Nanjing, China, October 2013, pp. 426–429
- [6] Zhang, Y., Ng, B. P.: 'MUSIC-Like DOA estimation without estimating the number of sources', *IEEE Trans. Signal Process.*, 2010, **58**, (3), pp. 1668–1676
- [7] Zeng, H., Ahmad, Z., Zhou, J., *et al.*: 'DOA estimation algorithm based on adaptive filtering in spatial domain', *China Commun.*, 2016, **13**, (12), pp. 49–58