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# Inflation dynamics in Uganda: a quantile regression approach

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## ABSTRACT

This paper considers the measurement of inflation persistence in Uganda and how this has changed over time within different quantiles. The measures of inflation include headline inflation and two measures of core inflation. The results suggest that while a unit root is found in many of the upper quantiles of headline inflation, there is evidence of mean reversion within the lower quantiles, which implies that large positive deviations influence the permanent behaviour of inflation. In addition, we find higher levels of persistence after 2006 and during the inflation-targeting period, where potential structural changes may have arisen within the regression quantiles.

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## 1 Introduction

One of the main objectives of most central banks, in both developed and developing countries, is to maintain low and stable rates of inflation. This applies to the Ugandan central bank, where the Bank of Uganda (BoU) is tasked with maintaining a rate of core inflation that is less than 5% over the medium-term.<sup>1</sup> To satisfy this objective, policymakers at the BoU need to consider the dynamic features of the inflation process and how it evolves over the medium- to long-term. These features would be largely influenced by the degree of inflation persistence, which is affected by various nominal price rigidities.

Measures of inflation are also usually included in multivariate macroeconomic models that are used by central banks for policymaking and forecasting purposes. The features of these models partially depend upon the characteristics of the data generation process for the inflation variable (Basher and Westerlund 2008). Therefore, it is important to consider whether shocks to the inflationary process are permanent or not, as this would affect the construction and evaluation of monetary policy rules (Culver and Papell 1997). In this paper, the persistence in the inflation rate is dependent upon the speed at which the inflation rate returns to its starting (or mean) level after a shock (c.f. De Oliveira and Pettrassi 2010; Wolters and Tillmann 2015). This would imply that a persistent rate of inflation would potentially increase the cost of monetary policy, in terms of the output and employment that would need to be sacrificed by the monetary authority to get inflation under control (Mishkin 2007). Inflation persistence thus plays an important role

in the formulation of monetary policy, since it provides the central bank with useful information about how the policy instruments should be employed to achieve the stipulated inflation target (Angeloni et al. 2006).

Uganda provides for an interesting setting for an analysis that considers the persistence in inflation, as core inflation was kept down to single digits during the early part of the twenty first century. However, between 2008 and 2012, the country experienced two heightened inflationary cycles in quick succession, where inflation rose to double digits. This could have altered the dynamics relating to the degree of persistence, since past inflation rates (intrinsic persistence) are regarded as the primary sources of inflation persistence (Fuhrer 2009). In addition, such an investigation would be of interest to policymakers in Uganda, who recently adopted an inflation targeting framework for the conduct of monetary policy. Such an analysis would also be of interest to researchers in developing countries and low-income countries (LICs), which have higher rates of inflation, when compared to most developed and advanced emerging economy countries.

A large body of literature on inflation persistence has examined the time-series properties of inflation by focusing only on the conditional mean of the process using standard time-series techniques, such as the procedure of Dickey and Fuller (1981). Under such a framework, a rejection of the null hypothesis of a unit root would imply that shocks dissipate and that the inflation rate would return to the equilibrium level after a period of time. Moreover, the exclusive focus on the conditional mean assumes that there will be a constant speed of adjustment, as the variable moves towards the mean, irrespective of the magnitude and direction of shocks that affect this variable. However, the inflation rate in most LICs, such as Uganda is frequently affected by shocks of differing magnitudes (large and small) and directions (positive and negative). As a result, the inflation rate in such countries may not be normally distributed. This would make estimates of persistence that are based on the conditional mean less informative.

Fortunately, the recent literature provides evidence of persistence in time-series based on the conditional distribution of a time-series using the Quantile Regression (QR) technique that was introduced by Koenker and Xiao (2004), and extended to the analysis of inflation persistence by Tsong and Lee (2011) and Wolters and Tillmann (2015). Such a detailed analysis of the level of persistence would allow us to investigate whether changes in persistence are harmonized across the various quantiles of the inflation rate and if anticipated changes in persistence are useful when seeking to explain changes in the distribution of the inflation rate (Wolters and Tillmann 2015).

The literature on inflation dynamics in Uganda has largely focused on the causes of inflation and very little has been published on the persistence of inflation. For example, Kabundi (2012) and Mawejeje and Lwanga (2015) use time-series analysis techniques to distinguish between the short-run and long-run causes of inflation. These studies suggest that both domestic and external factors are the main drivers of inflation in Uganda, over the short- and long-run. Some of the factors these studies identified include, domestic money growth, demand and supply shifts in the domestic agricultural sector, as well as developments in world food and energy prices (Kabundi 2012). It is also noted that Ugandan inflation may also be driven by structural factors, which have an unknown effect on the evolution of prices.

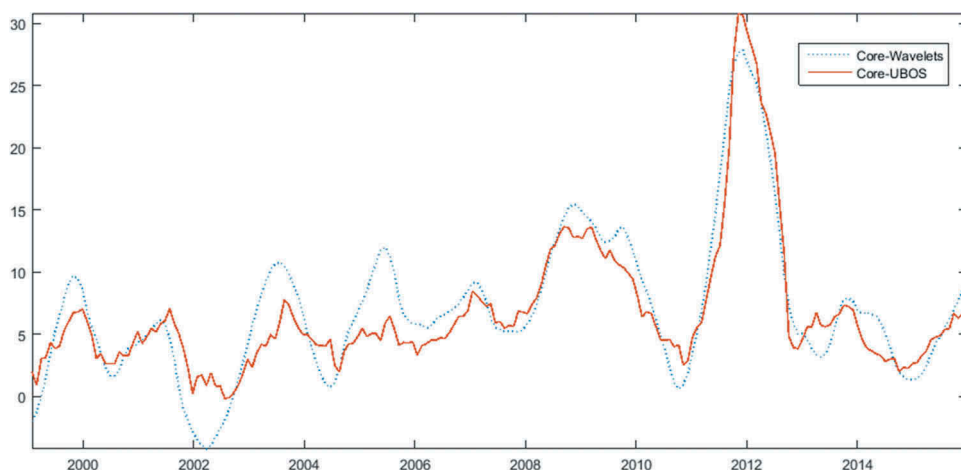
This paper also seeks to extend the application of time-varying unit root tests that may be applied with the aid of the quantile regression approach to investigate the varying degree of inflation persistence in a low-income country that has been subject to several large economic shocks. The first part of the paper considers the speed-of-adjustment (i.e. persistence) of the inflation rate in response to various macroeconomic shocks. To obtain time-varying persistence, the analysis makes use of a ten-year rolling-window. Persistence is also measured by the sum of the autoregressive coefficients (SARC), which is obtained from the Hansen (1999) grid bootstrap median unbiased estimator. In addition, we also calculate the half-lives (HLs) from the estimated SARC, to consider the robustness of the reported results. This analysis is applied to measures of headline and core inflation as both indicators are the two main measures of inflation in Uganda.<sup>2</sup> In the second half of the analysis, we consider the degree of persistence in an alternative measure of core inflation, which makes use of a wavelets transformation. This method for the measurement of core inflation is motivated by a recent study, which suggests that this technique may provide a useful measure of core inflation in South Africa (Du Plessis, Du Rand, and Kotzé 2015). This allows us to compare the potential usefulness of the two alternative measures of core inflation, which is the explicit target for monetary policy in Uganda.

The results of the study that was performed on headline inflation suggest that there is evidence of mean-reversion for the whole sample. However, when considering the different quantiles, we note that unit root behaviour is found in many of the higher quantiles, while there is stronger evidence of mean reversion in the lower quantiles. In addition, it is noted that the degree of persistence increases after 2006 and during the inflation-targeting period. These results are summarised over different periods of time, after we make use of the quantile regression method of Oka and Qu (2011) for detecting multiple structural breaks in the joint and individual quantiles. Similar findings are provided by both the official measure of core inflation and the alternative measure that is obtained from the wavelets transformation, where price shocks have had a permanent effect on inflation during the inflation-targeting period. Furthermore, it is also noted that the alternative measure of core inflation is less volatile and more correlated with headline inflation, which may imply that it could be more useful when formulating monetary policy.

The rest of the paper is structured as follows. [Section 2](#) briefly describes the observed features of the inflation process in Uganda over the last two decades. This is followed by [section 3](#), which reviews the related literature. The methodology employed and a description of the data are presented in [sections 4](#) and [5](#), respectively. [Section 6](#) then discusses the results before the conclusion is presented in the final section.

## 2 Stylised features of Ugandan inflation

[Figure 1](#) includes a number of different measures of inflation for Uganda over the period 2000 to 2015, where the average rate of headline and core inflation was 7.3% and 6.9%, respectively. However, during this period, Uganda experienced two episodes of increased inflationary pressure, in which core and headline inflation exceeded 10%. The first of these spikes was caused by a global food supply shock, when increased international food prices caused an upsurge in domestic inflation. This lasted from



**Figure 1.** Ugandan inflation rate January 2000 – December 2015.

April 2008 to September 2009 for the core measure, and from May 2008 to December 2009 for the headline measure. More recently, inflation spiked again, and price increases were rapid, with inflation peaking at almost 31% in October 2011. This second upsurge in Ugandan inflation was largely due to the effect of a global commodity price shock, coupled with a rapid depreciation in the value of the currency, and relatively low interest rates.

Between September 2012 and December 2015, the measure of core inflation averaged 4.8%, which is within the central bank target rate of 5%. Year-on-year headline inflation also averaged 5% over this period.

In July 2011, when the central bank adopted the inflation-targeting framework, consumer prices were on an upward trajectory, due to a series of supply shocks in agricultural production that resulted in food shortages and an increase in domestic food prices.<sup>3</sup> In addition, the sharp rise in the international crude oil price, and a depreciation in the exchange rate, also contributed to rising inflation. However, after the peak in October 2011, inflation has been on a downward trajectory. To a large degree this was driven by exchange rate appreciation over this period, coupled with a contractionary monetary policy phase. Since then, core inflation has remained close to the BoU target rate.

### 3 Literature review

The approaches that are used to measure persistence have evolved over time, following the development of several important time-series methods. In the early literature, many authors used conventional univariate models, where persistence was measured using scalar indicators such as the SARC, the largest autoregressive root, the half-life, and spectral density at the zero frequency (c.f. Andrews and Chen 1994; Marques 2004).<sup>4</sup> The use of such scalar measures of persistence was justified by the intuitive relationship between persistence and impulse response functions (IRFs).<sup>5</sup> Despite this relationship, IRFs could not be used as alternative measures of persistence, since they have the

potential to be infinite-length vectors, which makes it difficult to arrive at a single value to measure the degree of persistence.

The use of univariate models to measure inflation persistence was also challenged on the grounds that the mean (or equilibrium) inflation rate is presumed to be constant over time. This contradicts a large body of evidence which suggests that the dynamics of the inflation process change over time (c.f. Dossche and Everaert 2005; Gadzinski and Orlandi 2004; Levin, Natalucci, and Piger 2004). Moreover, some of the specific measures of persistence that are based on the univariate approach have also been criticised because they ignore potentially important information. For example, the measurement of persistence that relies on the largest autoregressive root has been challenged on the basis that it provides a very poor summary measure of the impulse response functions, because the shape of such a function would in fact depend on all the autoregressive roots, and not just the largest root (c.f. Andrews 1994 approximately; Pivetta 2007 persistence). Hence, ignoring the additional roots leads to measures of persistence that are less precise. Similarly, the measurement of persistence based on HLs has also been criticised as it does not identify changes in persistence over time, and the half-life for a highly persistent process is normally reported to be very large (Pivetta and Reis 2007).

Another strand of literature on inflation persistence argues that univariate models suffer from omitted variable bias, and suggest that multivariate models should be used to investigate persistence (c.f. Cogley and Sargent 2001; Pivetta and Reis 2007). According to proponents of this strand of the literature, the observed persistence in the inflation rate may be attributed to the evolution of the output gap, and omitting such information that is potentially important may result in biased and inconsistent estimates of persistence.<sup>6</sup>

A number of studies have also sought to model the time-varying nature of inflation persistence with both a univariate and multivariate framework. Such studies suggest that mean inflation rates have decreased in several countries, because of changes in the credibility of central banks over time. In addition, shifts in the mean inflation rates could be attributed to the emergence of more formal monetary policy frameworks (such as inflation-targeting) in many countries. This decrease in mean inflation rates may have altered the persistence in inflation. Schorfheide (2008), Caraianni (2009), and Dixon and Kara (2010), consider the possibility of shifts in the inflation mean (or equilibrium) before they suggest that the degree of inflation persistence may have changed over various periods of time. In addition, Lansing (2009), suggests that inflation-targeting resulted in lower rates of inflation, and this in turn distorts standard measures of persistence and volatility. Using a structural model that is based on the medium unbiased estimation technique, Benati (2008), investigates persistence for different periods in a number of European countries (pre and post-Monetary Union), the United Kingdom, Canada, Australia (before and after inflation-targeting), and the United States (before and after the Volcker disinflation period), and finds decreased inflation persistence in countries under inflation-targeting (where it is assumed that the United States adopted informal inflation-targeting). Similar results can be found in studies by Levin, Natalucci, and Piger (2004), Dossche and Everaert (2005), and Altissimo, Ehrmann, and Smets (2006). For instance, Dossche and Everaert (2005), suggest that the failure to account for shifts in mean inflation may result in a biased estimate of inflation persistence.

Clark (2006), investigate the level of inflation persistence in the United States by comparing persistence in the disaggregated and aggregated measures of consumer price inflation. He also examines whether inflation persistence has changed, based on a rolling window estimation procedure, similar to the approach employed by Stock and Watson (2003) and Pivetta and Reis (2007). The procedure he followed is also similar to the one used by Andrews and Chen (1994), where persistence is measured by the SARC that measures the cumulative long-run response of inflation to a shock (where the spectral density is close to zero). To obtain confidence intervals for the persistence estimates and the median unbiased estimates of persistence, Clark employs the grid bootstrap approach developed by Hansen (1999). The results from Clark's paper reveal that, when mean inflation is assumed to remain unchanged within samples, the average persistence in disaggregated inflation is consistently below aggregate persistence. Furthermore, the aggregated measure of persistence was reduced, marginally. The model for disaggregate data showed that inflation persistence declined during the same period. This suggests that there has been a mean shift in inflation persistence for both the aggregate and disaggregate measure of inflation. These results continue to hold after accounting for changes in the mean rates of inflation.

Several studies suggest that structural breaks may lead to an overestimation of the level of persistence. To alleviate this problem, they propose models that include structural breaks (c.f. Perron 1990; Levin and Piger 2002). For instance, Gadzinski and Orlandi (2004), investigate inflation persistence in the euro area and the United States, using an autoregressive process that allows for structural breaks, and a time varying mean, and find decreased inflation persistence after 1984. In addition, they find that persistence in the euro area is comparable to that of the United States. By employing a non-parametric approach, Belbute, Massala, and Delgado (2015), examine inflation persistence in Angola and find mean reversion when a structural break is included. In addition, they find a low level of inflation persistence throughout the sample. Similarly, Balcilar, Gupta, and Jooste (2016), makes use of a Markov-Switching autoregressive fractionally integrated moving average model, to test for the presence of a long memory amidst changes in inflationary regimes, and find that the level of volatility and persistence in South African inflation increases during periods of high inflation. Cuestas and Mourelle (2009), apply a logistic smooth transition autoregressive (LSTAR) model to investigate inflation persistence in selected African countries, and find that there is a relatively low level of persistence in most of the countries sampled. In addition, they find that the estimated models are stable, suggesting that in the absence of exogenous shocks, inflation would possibly stay within a single regime.

A related approach that investigates time-varying persistence, and which will be followed in this paper, can be found in Tsong and Lee (2011) and Wolters and Tillmann (2015). This approach explores uses the quantile regression technique that was proposed by (Koenker and Xiao 2004) to measure the time variation in inflationary persistence at different quantiles. Tsong and Lee (2011), apply this framework to investigate the dynamics of inflation in twelve OECD countries, and find that inflation rates are not only mean-reverting but also asymmetric. Wolters and Tillmann (2015) have also applied this technique on post-war inflation data for the United States, where they identify a number of break points at various quantiles during the 1980s. They attribute these to the time-varying nature of the mean rate of inflation.

## 4 Empirical methodology

### 4.1 Univariate autoregressive model

One of the early approaches to measure persistence in a time-series involved using univariate autoregressive models. The model specification for the standard univariate AR ( $q$ ) process for inflation, could be expressed as:

$$\pi_t = \alpha + \sum_{j=1}^q \beta_j \pi_{t-j} + \varepsilon_t, \quad (1)$$

where  $\pi_t$  is the measure of inflation (i.e. either core or headline inflation measured at a monthly or quarterly frequency),  $\alpha$  is the intercept term and  $\varepsilon_t$  is an independent and identically distributed (i.i.d) innovation to the inflation rate with mean zero and a constant variance,  $\sigma^2$ . The persistence in the inflation rate is then measured using the scalar measure of the SARC ( $\rho = \sum_{j=1}^q \beta_j$ ) in equation (1). The justification for this scalar measure of persistence is discussed in Andrews and Chen (1994), who argue that the long-run persistence property in a time-series can be displayed with the aid of an impulse response function. Consequently, Andrews and Chen (1994), suggest that it is appropriate to use the SARC to capture persistence, because there is a monotonic relationship between  $\rho$  and the cumulative impulse response function for future values of inflation ( $\pi_{t+j}$ ) due to a shock to the inflation rate,  $\varepsilon_t$ . To capture the sum of the autocorrelation coefficients for the inflation rate, equation (1) can be rewritten as:

$$\pi_t = \alpha + \rho \pi_{t-1} + \sum_{j=1}^{q-1} \delta_j \Delta \pi_{t-j} + \varepsilon_t. \quad (2)$$

Equation (2) corresponds to the popular Augmented Dickey Fuller (ADF) regression by Dickey and Fuller (1979, 1981), which is used to determine whether a time-series process is stationary, where the change in the inflation rate between two periods is expressed as,  $\Delta \pi_t = \pi_t - \pi_{t-1}$ . The value of  $\rho$  in equation (2) contains important information about the inflation process, such that when  $|\rho| = 1$ , the inflation rate contains a unit root (i.e. it is a random walk process). A variable with such a characteristic is said to display infinite persistence. In contrast, when the true value of the coefficient is  $|\rho| < 1$ , the inflation rate would exhibit mean reverting characteristics following a shock and the process is said to be stationary.

When the autoregressive process has a unit root, then the estimate obtained for this coefficient in equation (2) is subject to downward bias when classical techniques are used. To resolve this problem, Andrews and Chen (1994) and Hansen (1999) suggest that the median unbiased estimate of  $\rho$  should be used, which is derived from a bootstrap procedure. This bootstrap procedure could also be used to construct confidence intervals that relate to the coefficient estimate. In this paper, we follow Tsong and Lee (2011) and Wolters and Tillmann (2015) who make use of this approach to derive estimates for the conditional quantiles.

## 4.2 Quantile autoregressive models

Quantile autoregressive (QAR henceforth) models are derived from univariate autoregressive models, which may be used to investigate persistence when one would want to consider potential differences that may arise from different parts of the data. The application of the QAR method to unit root tests was introduced in Koenker and Xiao (2004). This literature was later extended to the analysis of inflation persistence by Tsong and Lee (2011), and recently by Wolters and Tillmann (2015). This framework makes use of quantiles to describe the distribution of coefficients according to the proportion of observations that may be classified by the respective quantiles of the data. Within this context, this approach allows us to explore unit root characteristics across the different quantiles. It also allows us to measure the speed of adjustment of the time-series back to its mean level, following shocks that have different magnitudes and signs. Moreover, since the unit root test in the QAR framework is performed at individual quantiles, we avoid making the strong assumption that the time-series is normally distributed, which is the case in other standard unit root tests, such as Dickey and Fuller (1979, 1981).

Therefore, to obtain the  $QAR(q)$  from equation (2), we define the  $\tau^{th}$  quantile as  $q_\tau(\pi_t|\pi_{t-1}, \dots, \pi_{t-q})$  and the probability that the conditional inflation rate will lie above or below the  $\tau^{th}$  quantile is given as  $\tau$  and  $1 - \tau$ , respectively. The resulting  $QAR(q)$  can then be expressed as:

$$q_\tau(\pi_t|\pi_{t-1}, \dots, \pi_{t-q}) = \alpha(\tau) + \rho(\tau)\pi_{t-1} + \sum_{j=1}^{q-1} \delta_j \Delta\pi_{t-j}. \quad (3)$$

where,  $\rho(\tau)$  is the measure of the degree of persistence in the inflation rate, conditional on all the lagged inflation rates  $(\pi_{t-1}, \dots, \pi_{t-q})$ . This procedure requires that the  $\tau^{th}$  quantile be suitably identified.

A second measure of persistence, which we will also investigate, may be derived from the measures of HLs in each quantile. The HLs measure the number of periods in which the inflation rate remains above half of its initial level, following a shock to the inflation rate. After  $\hat{\rho}(\tau)$  is obtained, we calculate the HLs using the formula  $\log(0.5)/\log(\hat{\rho}(\tau))$ .

To estimate the parameters in equation (2), we must choose the order of the autoregression (i.e. the lag length,  $q$ , in the QAR). Due to the nature of the data that is utilized in this paper, we assume a lag length of 4 for the quarterly data and a lag length of 12 for the monthly data, as this would correspond to a lag of one year.

## 4.3 QAR structural break test

To consider the possibility that the data sample may include multiple structural breaks during unknown periods of time, we make use of the methodology of Oka and Qu (2011) to test for the existence of one or multiple breaks in each of the conditional quantile functions. This framework may be applied to the  $QAR(\rho)$  model of Koenker and Xiao (2006), where an  $SQ_\tau$  test is used to identify a structural change in quantile  $\tau$  and the  $DQ$  test is used to identify a structural change in quantiles over a particular interval. These statistics make use of null ( $H_0$ ) and alternative ( $H_1$ ) hypotheses:

$$H_0 : \beta_i(\tau) = \beta_0(\tau) \text{ for all } i$$

$$H_1 : \beta_i(\tau) = \begin{cases} \beta_1(\tau) & \text{for } i = 1, 2, \dots, n_1 \\ \beta_2(\tau) & \text{for } i = n_1 + 1, \dots, T \end{cases}$$

where  $\beta(\tau)$  are the unknown parameters that are quantile dependent. For the sample size of  $[\lambda T]$ , with  $0 \leq \lambda \leq 1$ , the  $SQ_\tau$  test may be expressed as:

$$SQ_\tau = \sup_{\lambda \in [0,1]} \left\| (\tau(1-\tau))^{-1/2} \left[ H_{\lambda,T}(\hat{\beta}(\tau)) - \lambda H_{1,T}(\hat{\beta}(\tau)) \right] \right\|_\infty$$

where

$$H_{\lambda,T}(\hat{\beta}(\tau)) = \left( \sum_{t=1}^T x_t x_t' \right)^{-1/2} \sum_{t=1}^{\lambda T} x_t \psi_\tau(y_t - x_t' \hat{\beta}(\tau))$$

and the corresponding dependent and independent variables are denoted  $y_t$  and  $x_t$ , while  $\psi_\tau(u) = \tau - 1(u < 0)$ . Similarly, the  $DQ$  test may be defined as:

$$DQ = \sup_{\tau \in \mathcal{T}_\omega} \sup_{\lambda \in [0,1]} \left\| H_{\lambda,T}(\hat{\beta}(\tau)) - \lambda H_{1,T}(\hat{\beta}(\tau)) \right\|_\infty$$

where  $\mathcal{T}_\omega$  denotes the joint quantile intervals over which the test is performed. These tests are asymptotically free of nuisance parameters and critical values are provided in Qu (2008).

#### 4.4 QAR unit root test

To test the time-series properties of  $\pi_t$  (i.e  $H_0 : \rho(\tau) = 1$ ) within the  $\tau$ -th quantile, Koenker and Xiao (2004), suggests a testing statistic that is based on the following  $t$ -statistics:

$$t_n(\tau) = \frac{\hat{g}(G^{-1}(\tau))}{\sqrt{\tau(1-\tau)}} \left( \pi_{-1}' M_X \pi_{-1} \right)^{1/2} (\hat{\rho}(\tau) - 1) \quad (4)$$

where the function  $\hat{g}(G^{-1}(\tau))$  corresponds to the consistent estimator of  $g(G^{-1}(\tau))$ , while  $g$  and  $G$  denote the density and distribution function of  $\varepsilon_t$  in equation (2). In this case,  $M_X$  denotes the projection matrix for the space orthogonal to  $X = (1, \Delta\pi_{t-1}, \dots, \Delta\pi_{t-k+1})$ .

This test statistic allows us to explore the unit root behaviour of the inflation rate in the different quantiles, to capture the dynamics of the shocks that affect the inflation rate at different quantiles of the distribution. Another approach involves assessing the unit root property over a range of quantiles. To achieve this, Koenker and Xiao (2004) suggest the use of the quantile Kolmogorov-Smirnov (QKS) test, which is expressed as:

$$QKS = \sup |t_n(\tau)| \quad (5)$$

In this setting,  $t_n(\tau)$  represents the  $t$  ratio statistics as defined in equation (4). The implementation of this test involves first calculating  $t_n(\tau)$  at  $\tau \in \Gamma$ , which is used to compute the QKS test, by calculating the maximum over  $\Gamma$ .

The limiting distribution of the  $t_n(\tau)$  and *QKS* tests are non-standard, and depend on nuisance parameters. One approach used to estimate nuisance parameters involves applying the re-sampling procedure of Koenker and Xiao (2004). As an alternative, Galvao (2009) proposes the use of a simulation strategy to derive these nuisance parameters.

#### 4.5 Wavelets estimate of core inflation

Graps (1995) describes wavelets as mathematical functions that are used to decompose data into different frequency components, which can then be used to study the behaviour of each component in a resolution that is matched to its scale. The early literature on the decomposition of macroeconomic data that contains a unit root includes the methodologies developed by Hodrick and Prescott (1997), Baxter and King (1999), and Christiano and Fitzgerald (2003). These techniques may be implemented with the aid of a Fourier transformation, which assume that the properties of the underlying variable do not change. Since the wavelet decomposition is cast in the time-frequency domain, which does not encounter a loss of the time support, it may allow for changes in the properties of the underlying data over time. In addition, it would also allow for varying degrees of integration.

This technique makes use of a father wavelet,  $\phi$ , which are also referred to as scaling functions that are used to represent the smooth baseline trend. The mother wavelets,  $\psi$ , are then used to represent the deviations from the trending component. Therefore, a time-series variable, which in this case relates to headline inflation, can be transformed with the aid of multi-resolution techniques that comprise of both father and mother wavelets, using the expression;

$$\pi_t = \sum_{k=1}^{N_j} v_{j,k} \phi_{j,k,t} + \sum_{j=1}^J \sum_{k=1}^{N_j} w_{j,k} \psi_{j,k,t}. \quad (6)$$

where  $v_{j,k}$  and  $w_{j,k}$  are the coefficients and  $N_j$  are the number of coefficients in the  $j - th$  scale. In this analysis, we follow Du Plessis, Du Rand, and Kotzé (2015) and make use of the Daubechies smoothed wavelets functions.<sup>7</sup>

## 5 Data

The analysis makes use of year-on-year monthly and quarterly data for changes in the headline Consumer Price Index (CPI) and core CPI. The core CPI is the component of headline CPI that excludes prices related to food crops, electricity, fuel, and metered water. Core CPI currently constitutes 82.4% of headline CPI. The analysis makes use of data from July 1993 to December 2015, for the headline inflation (270 observations), and from July 1997 to December 2015, for core inflation (210 observations). The start and end dates are determined by data availability. This data sample corresponds to quarterly data periods 1993Q3 to 2015Q4 (90 observations) and 1998Q3 to 2015Q4 (70 observations) for the headline and core inflation rates, respectively. The year-on-year monthly headline and core inflation rates are computed by taking the difference of the natural logarithmic of CPI for a given month in the current year and the same month from the

previous year. The year-on-year quarterly inflation rates are calculated as the difference in the natural logarithmic of CPI for a given quarter in the current year and the same quarter from the previous year. The monthly time-series data was obtained from the UBOS website and the quarterly time-series was obtained by taking the average index value for the three months in a quarter.

The inflationary process in Uganda may have been influenced by changes in the monetary policy framework and the effects of various shocks that may be due to the global financial crisis and the spike in commodity prices. Therefore, we subject the various measures of the inflationary process to a quantile regression structural break test, which is described in section 4.3. To implement this analysis to the univariate autoregressive model we make use of seven equally spaced quantiles,  $\tau = 0.2, 0.3, 0.4, 0.5, 0.7, 0.8$ , which are chosen to examine the possibility of a joint structural break over multiple quantiles and the dispersion of the conditional distribution. In all of these tests, we set the maximum number of structural breaks to three, which would appear to be appropriate after considering the results of this analysis.

The results for the  $DQ$  test statistics, which summarise the joint of the seven quantiles are summarised in Table 1. For each of these measures of the inflationary process, we note that the highest  $DQ(1|0)$  statistic is 0.76, which is less than the critical value at the 5% level of significance. Therefore, we conclude that when considering the seven quantiles as part of a joint analysis, there appears to be no structural break present in any of the respective measures. In this case it would not be necessary to test for the existence of a second or third structural break.

Table 2 contains the results of analysis for the individual quantiles that report on the  $SQ_\tau$  test statistics. Here we note that for headline inflation that is measured on a quarterly basis, there would appear to be a structural break in the 70% and 80% quantiles. In the case of the 70% quantile, we note that the estimated break date is 2008Q1, with a 95% confidence interval that extends from 2000Q2 to 2013Q1. Similarly, the estimated break date in the 80% quantile is 1999Q3, with a confidence interval over 1997Q1 to 1999Q2. In both these cases, test statistics for a second structural break are lower than the respective critical values. When considering the results for monthly headline inflation, there would appear to be a structural break in the 20% quantile around 2002M4, prior to the global financial crisis. This quantile would possibly pertain to those elements of inflation that are not particularly persistent and as such are of less interest in this particular study.

For both the monthly and quarterly measures of core inflation we identify potential structural breaks in the 80% quantile. For the quarterly measure there are two estimated break dates that are significant at the 5% level. These arise in 2007Q3 and 2011Q4. In the first case the confidence interval extends over the period 2007Q2 to 2010Q1, while in

**Table 1.** Structural breaks over joint quantiles.

	Headline(q)	Headline(m)	Core(q)	Core(m)
$DQ(1 0)$	0.76	0.69	0.62	0.71
Critical value (5%)	0.91	0.91	0.91	0.91
Break date	–	–	–	–

m & q refers to monthly and quarterly series respectively.

\* denotes significance at the 5% level.

**Table 2.** Structural breaks in individual quantiles.

	Quantiles						
	0.2	0.3	0.4	0.5	0.6	0.7	0.8
<i>Headline (q)</i>							
SQ(1 0)	1.24	1.1	0.9	0.89	0.92	1.61*	1.92*
Critical value (5%)	1.52	1.52	1.52	1.52	1.52	1.52	1.52
Break date	–	–	–	–	–	2008:03	1997:09
Conf. Interval (95%)	–	–	–	–	–	[00:06, 13:03]	[97:03, 99:06]
SQ(2 1)	–	–	–	–	–	1.54	1.26
Critical value (5%)	–	–	–	–	–	1.62	1.62
Break date	–	–	–	–	–	–	–
<i>Headline (m)</i>							
SQ(1 0)	1.65*	1.19	0.83	0.59	0.88	1.25	1.13
Critical value (5%)	1.52	1.52	1.52	1.52	1.52	1.52	1.52
Break date	2002:04	–	–	–	–	–	–
Conf. Interval (95%)	[95:04, 05:05]	–	–	–	–	–	–
SQ(2 1)	1.09	–	–	–	–	–	–
Critical value (5%)	1.62	–	–	–	–	–	–
Break date	–	–	–	–	–	–	–
<i>Core (q)</i>							
SQ(1 0)	1.27	1.04	0	0.64	1.25	1	1.55*
Critical value (5%)	1.52	1.52	1.52	1.52	1.52	1.52	1.52
Break date	–	–	–	–	–	–	2007:09
Conf. Interval (95%)	–	–	–	–	–	–	[07:06, 10:03]
SQ(2 1)	–	–	–	–	–	–	2.17*
Critical value (5%)	–	–	–	–	–	–	1.62
Break date	–	–	–	–	–	–	2011:12
Conf. Interval (95%)	–	–	–	–	–	–	[06:09, 12:12]
SQ(3 2)	–	–	–	–	–	–	0.95
Critical value (5%)	–	–	–	–	–	–	1.69
Break date	–	–	–	–	–	–	–
<i>Core (m)</i>							
SQ(1 0)	1.43	1.31	1.12	1.14	0.98	1.37	1.69*
Critical value (5%)	1.52	1.52	1.52	1.52	1.52	1.52	1.52
Break date	–	–	–	–	–	–	2007:12
Conf. Interval (95%)	–	–	–	–	–	–	[05:08, 10:04]
SQ(2 1)	–	–	–	–	–	–	1.42
Critical value (5%)	–	–	–	–	–	–	1.62
Break date	–	–	–	–	–	–	–

m & q refers to monthly and quarterly series respectively.

\* denotes significance at the 5% level.

the second it ranges from 2006Q3 to 2012Q4. Similarly for monthly core inflation, we note that the single estimated break date is 2007M12, with a 95% confidence interval that extends from 2005M8 to 2010M4.

To summarise these results, while we find that there is no evidence of structural break for the seven joint quantiles, the results suggest presence of structural break in some of the individual quantiles for each of the measures of inflation. Such a break may have occurred during the period prior to the global financial crisis. In addition, the commodity price shock that arose in 2011 may also have given rise to a structural break.

To consider the general properties of the data, Table 3 presents the key descriptive statistics for the two measures of inflation. They represent the first four sample moments and the Jarque-Bera (JB) normality test for the respective inflation rates. In addition, since this paper is also concerned with the degree of time-varying persistence, the summary statistics are reported for two sub-samples, after splitting the entire dataset in 2007. The results of the first moment show that the mean inflation rate for both

**Table 3.** Summary Statistics for headline and core inflation rates.

Variable	Sample	Mean	Std. dev.	Skewness	Kurtosis	JB Statistic
Headline(q)	1993–2015	6.89	5.70	1.04	5.39	37.96**(0.000)
Headline(m)		6.89	5.74	1.08	5.69	133.21**(0.000)
Core(q)	1998–2015	6.61	5.41	2.40	9.57	193.40**(0.000)
Core(m)		6.62	5.48	2.42	9.72	600.71**(0.000)
Headline(q)	1993–2006	5.31	4.41	−0.25	2.63	0.85(0.652)
Headline(m)		5.31	4.37	−0.33	2.73	3.46(0.177)
Core(q)	1998–2006	4.06	1.85	−0.36	2.45	1.17(0.556)
Core(m)		4.06	1.92	−0.33	2.67	2.36(0.307)
Headline(q)	2007–2015	9.27	6.59	1.21	4.04	10.45**(0.005)
Headline(m)		9.27	6.67	1.23	4.20	33.67**(0.000)
Core(q)	2007–2015	9.03	6.49	1.74	5.61	28.31**(0.000)
Core(m)		9.03	6.56	1.76	5.71	88.77**(0.000)

JB stat. refers to the Jarque-Bera normality test and this test is  $\chi^2$  distributed asymptotically.

m & q refers to monthly and quarterly series respectively.

\*\* denotes significance at the 5% level.

headline and core inflation have increased during the more recent period. Specifically, the mean inflation rate for the sub-sample 2007–2015 increased to 9.27% and 9.03% for the quarterly and monthly measures, respectively. This is off a relatively low base, as the respective mean quarterly and monthly rates were 5.31% and 4.06% for the sub-sample prior to 2007. Inflation remained high in the period following the global financial crisis, on account of domestic currency depreciation and monetary expansion.

Within each of the sub-samples, it is also worth noting that the mean of the headline inflation rate is higher than the mean of the core inflation rate. In addition, the standard deviations in each sub-sample suggest that core inflation has successfully removed those components that are considered to be more volatile.

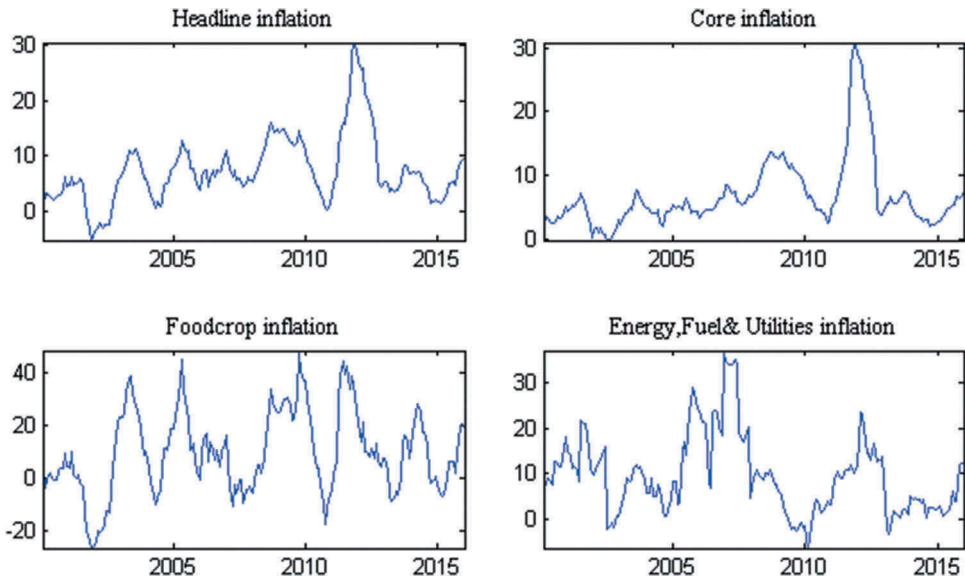
The correlation coefficients between headline and core inflation for the two sub-samples are lower for the earlier sub-sample at 0.84 and 0.85, for the respective monthly and quarterly rates. These measures of correlation increased to 0.97 and 0.96, for the second sub-sample. Note also that the second moment also increases in the second sub-sample. This behaviour is consistent with the literature in this field, where Okun (1971), Davis and Kanago (1998), Daal, Naka, and Sanchez (2005), and Tsong and Lee (2011), suggest that in most cases a high rate of inflation moves together with higher levels of inflation uncertainty.

Then lastly, the JB test for the full sample and the sub-sample after 2006 strongly rejects the null hypothesis of normality, given the extremely small  $p$ -values. This supports the decision to model persistence with the aid of a quantile regression approach that could describe the behaviour that may arise at different parts of the distribution.

## 6 Results

### 6.1 Time-varying persistence

We make use of a ten-year rolling-window to explore the behaviour of inflation persistence in the conditional mean and median of the two measures of inflation. The results of the rolling-window coefficient estimates for persistence (dotted line is the conditional mean and solid line is the conditional median) along with the 95% bootstrap confidence



**Figure 2.** Monthly inflation rate persistence.

Notes: The graphs plot the estimates of  $\rho$  in the 10-year rolling window where the conditional mean and median are represented by the dotted and solid lines, respectively. The gray area represents the 95% bootstrap confidence intervals.

intervals (grey area) for the monthly inflation rates are displayed in [Figure 2](#). The corresponding graph for the quarterly measures is contained in the appendix, as [Figure A1](#). The graphs show that headline inflation was largely mean-reverting for the bulk of the sample, implying that shocks to headline inflation were short-lived. However, the degree of persistence in headline inflation increased towards the end of the sample. This behaviour is more pronounced in the quarterly inflation rate. To be more specific, the persistence in the monthly headline inflation rate increased from about 0.8, at the start of the study period, to about 0.95 at the end of the study period. Similarly, the persistence in the quarterly headline inflation rate also increased during the same period, from about 0.5 to 0.9.

Turning to the core inflation rate, the graph suggests that the persistence in core inflation was somewhat lower at the end of the study period. Once again, this trajectory is more pronounced in the quarterly measure. The results in [Figure 2](#) also suggest that persistence in the monthly core inflation rates remained close to unitary after 2007, showing occasional signs that the process may be characterised as a unit root.

The observed behaviour in the persistence towards the end of the sample is contrary to some of the findings in the literature for most advanced and some middle-income countries, which find declining levels of inflation persistence during the more recent period. However, these findings are consistent with Oliveira and Petrassi (2014), who find increased inflation rate persistence in several emerging countries during the recent period. The increased inflation rate persistence in Uganda during this period could be attributed to the effect of the global commodity price boom before the recent financial crisis. In addition, it may also have been influenced by the sharp rise in domestic food and energy prices, as well as the deterioration of the exchange rate.<sup>8</sup>

## 6.2 Quantile regression results

We now explore the dynamic behaviour of inflationary persistence using a quantile regression, since the normality test established asymmetry in the inflation rates in Uganda. Our analysis is based on the pioneering work of Koenker and Xiao (2004), who proposed that, when there is significant evidence of non-normality in an economic variable, a quantile regression approach would provide superior estimates for the level of persistence. The results of the QAR unit root tests for the individual quantiles and the QKS tests are reported in Table 4 for the data that is measured at a monthly frequency.<sup>9</sup> The table reports the results for the full sample and the respective sub-samples that are based on the summary statistics, and the start of inflation-targeting regime. However, for the quarterly series we do not investigate persistence during the inflation-targeting period, as the sample size is too small. The results from the sub-samples are used as a robustness check as a structural break in the data may affect the measure of persistence.

When considering the results of the QKS tests for the whole sample, which is used to quantify mean-reverting behaviour of each measure of inflation, we find overwhelming evidence in support of mean-reversion in both monthly and quarterly headline and core inflation rates. However, the detailed results which consider the behaviour of each measure of inflation in the different quantiles, reveal significant differences in the estimated values of the autoregressive coefficient  $\rho(\tau)$  across quantiles. For instance, the results reveal that the estimates of  $\rho(\tau)$  in the bottom quantiles are well below unity, and have small t-values, which rejects the unit root null at the 5% significance level. However, the estimates of  $\rho(\tau)$  in the top quantiles (especially, at the 60%, 70% and 80% quantiles) are close to or slightly larger than unity, and the t-values do not reject the unit root null at the 5% significance level. Thus, the results suggest that in the upper quantiles, the inflation rates contain a unit root, while in others, shocks to inflation are short-lived, and the inflation rate would return to its mean value.

From the estimated values of  $\rho(\tau)$ , we calculate the HLs in each quantile for the two inflation series over the whole sample. The results suggest that the HLs in the lower quantiles are generally smaller, they become larger (occasionally infinite) in the upper quantiles. For example, below the 50% quantile, the HLs for the quarterly inflation rates range from 1.5 quarters (headline inflation) to 3.01 quarters (core inflation), implying that in the lower quantiles when the inflation rate is hit by a large shock, it could quickly return to its long-run level. However, when considering the estimates that are above the median quantile, the HLs for the quarterly inflation rates go as high as infinity for the core inflation measure, implying that for such quantiles the inflation rate may never return to its long-run equilibrium. These results confirm that persistence in monthly and quarterly inflation rates is asymmetric, in the sense that when inflation exhibits a large negative deviation from its mean, it is short lived. However, a large positive deviation from its mean would influence the permanent behaviour of inflation. This has important implications for monetary policy in Uganda.

## 6.3 Structural breaks in persistence

Failure to account for structural breaks in unit root tests could lead to an over exaggeration of the persistence of a time-series process (c.f. Perron 1990; Perron and Vogelsang

**Table 4.** Quantile unit root tests for monthly inflation rate.

Variable	Period	$\tau$	0.20	0.30	0.40	0.50	0.60	0.70	0.80
Headline	1993M7-2015M12	$\hat{\rho}(\tau)$	0.90	0.92	0.89	0.92	0.92	0.94	0.94
		Unit root	no	yes	no	no	no	yes	yes
		$t$ -stat	-2.91	-2.57	-3.56	-2.80	-3.17	-2.32	-1.81
		critical value	-2.52	-2.57	-2.56	-2.59	-2.62	-2.55	-2.59
		Half-lives	6.55	8.01	6.19	7.97	7.81	10.42	12.05
		KS-test	unit root: no	QKS = 3.56 cv = 2.90					
	1993M7-2006M12	$\hat{\rho}(\tau)$	0.82	0.82	0.78	0.81	0.88	0.91	0.98
		Unit root	yes	yes	no	no	yes	yes	yes
		$t$ -stat	-2.69	-2.64	-3.78	-3.07	-2.11	-1.24	-0.29
		critical value	-2.74	-2.68	-2.59	-2.53	-2.39	-2.48	-2.44
Half-lives		3.57	3.47	2.80	3.38	5.22	7.15	35.67	
KS-test		unit root: no	QKS = 3.78 cv = 2.90						
2007M1-2011M6	$\hat{\rho}(\tau)$	0.64	0.73	0.77	0.73	0.82	0.87	0.88	
	Unit root	no	no	yes	yes	yes	yes	yes	
	$t$ -stat	-8.18	-2.72	-2.18	-2.16	-1.20	-0.78	-0.93	
	critical value	-2.17	-2.40	-2.38	-2.26	-2.71	-2.53	-2.25	
	Half-lives	1.54	2.24	2.63	2.24	3.51	5.07	5.28	
	KS-test	unit root: no	QKS = 8.18 cv = 2.89						
2011M7-2015M12	$\hat{\rho}(\tau)$	0.77	0.90	0.93	0.90	0.96	0.96	0.97	
	Unit root	no	yes	yes	yes	yes	yes	yes	
	$t$ -stat	-2.92	-1.21	-0.80	-1.11	-0.53	-0.46	-0.47	
	critical value	-2.12	-2.14	-2.12	-2.35	-2.24	-2.46	-2.54	
	Half-lives	2.70	6.73	9.75	6.69	16.15	18.79	20.86	
	KS-test	unit root: no	QKS = 2.92 cv = 2.88						
Core	1998M7-2015M12	$\hat{\rho}(\tau)$	0.93	0.95	0.93	0.94	0.94	0.96	0.98
		Unit root	yes	no	no	no	no	yes	yes
		$t$ -stat	-2.28	-2.67	-3.85	-3.28	-2.80	-1.82	-0.69
		critical value	-2.43	-2.51	-2.55	-2.59	-2.63	-2.71	-2.62
		Half-lives	10.31	12.37	9.72	11.18	12.16	16.80	39.27
		KS-test	unit root: no	QKS = 3.85 cv = 2.89					
	1998M7-2006M12	$\hat{\rho}(\tau)$	0.64	0.69	0.79	0.79	0.81	0.82	0.77
		Unit root	yes	yes	yes	yes	yes	yes	yes
		$t$ -stat	-2.37	-1.78	-1.74	-1.70	-1.75	-1.54	-1.99
		critical value	-2.75	-2.78	-2.78	-2.73	-2.61	-2.47	-2.26
Half-lives		1.56	1.89	2.99	2.96	3.25	3.50	2.72	
KS-test		unit root: yes	QKS = 2.37 cv = 2.88						
2007M1-2011M6	$\hat{\rho}(\tau)$	0.76	0.77	0.84	0.82	0.82	0.78	0.83	
	Unit root	no	no	yes	yes	yes	yes	yes	
	$t$ -stat	-2.92	-2.68	-1.64	-1.64	-1.88	-2.13	-1.57	
	critical value	-2.14	-2.37	-2.41	-2.68	-2.60	-2.47	-2.18	
	Half-lives	2.55	2.63	3.96	3.59	3.41	2.75	3.81	
	KS-test	unit root: no	QKS = 3.02 cv = 2.88						
2011M7-2015M12	$\hat{\rho}(\tau)$	0.91	0.94	0.97	0.96	0.96	0.97	1.01	
	Unit root	no	yes	yes	yes	yes	yes	yes	
	$t$ -stat	-2.88	-1.80	-1.05	-1.11	-1.14	-0.70	0.27	
	critical value	-2.12	-2.28	-2.45	-2.30	-2.54	-2.30	-2.34	
	Half-lives	7.52	11.45	21.90	18.22	19.40	23.25	$\infty$	
	KS-test	unit root: yes	QKS = 2.88 cv = 2.91						

Critical value (cv) at the 5% significance level. QKS statistics are calculated for the entire sample. We reject the null when the calculated test statistics is less than the critical value. Infinity HLs mean  $\hat{\rho}(\tau)$  is larger than unity.

1992). Uganda adopted inflation-targeting in 2011 which could have caused a shift in the mean rate of inflation. In addition, the results of the normality tests that were presented earlier suggest a possible shift in the mean that may have occurred during 2007. To quantify the effect of possible structural breaks on persistence, we use the

results in [Table 4](#), to compare the persistence over different subsamples with the estimates that were provided for the full sample. These results suggest that the QKS test provides overwhelming evidence in favour of mean-reverting headline inflation for all sub-samples, which is similar to the results that are based on the full sample. However, there are substantial differences in the values of  $\hat{\rho}(\tau)$  within the individual quantiles for the different sub-samples. For instance, the degree of persistence is generally higher in all the quantiles during the period 2007Q1-2015Q4. Moreover, within each sub-sample, the persistence of the headline inflation rate is still asymmetric. With respect to the HLs, they are mostly lower in the sub-samples.

Secondly, turning to the core inflation rate, in contrast to the results based on the whole sample, the QKS test provides evidence of a unit root during some periods (1998Q3-2006Q4, 1998M7- 2006M12, and 2011M7-2015M12). While the detailed results for the specific quantiles reveal substantial differences in the estimated levels of persistence for the different sub-samples, when compared to the full sample. For example, the degree of persistence is generally reduced in most quantiles, especially in the lower ones, during 2007Q1-2015Q4. However, persistence in the core inflation rate is still asymmetric within each sub-sample. While the HLs are mostly lower in the sub-samples, relative to the entire sample.

In summary, the behaviour of persistence in the subsamples varies greatly from that of the full sample, and the detailed results show substantial differences between the sub-samples and entire sample. This suggests that the characteristics of persistence depends on the structure of the inflation rates at specific periods. Notably, the degree of persistence generally increased after 2006.<sup>10</sup> These results identify periods that are largely consistent with the estimated break points in many of the upper quantiles, which were identified in [section 5](#).

#### **6.4 Quantile regression results for the inflation-targeting period**

It has been suggested that in several countries, inflation persistence decreased after the implementation of an inflation-targeting monetary policy regime (c.f. Bratsiotis, Madsen, and Martin 2015; Levin, Natalucci, and Piger 2004; Benati 2008; Rangasamy 2009; Gerlach and Tillmann 2012). In this section, we quantify whether the persistence in Ugandan inflation has been lower during the inflation-targeting period. We compare the persistence estimates from [Table 4](#) during the inflation-targeting period (2011M7-2015M12) with the sub-samples prior to the implementation of this framework, and note the following. First, similar to the sub-samples that pre-dated inflation-targeting, the QKS test suggests that headline inflation has been mean-reverting during the inflation-targeting period. However, though the detailed estimates of the degree of persistence are substantially different for the periods before and after inflation-targeting, the difference between the lower and upper quantiles remains. More specifically, in most quantiles the degree of persistence for headline inflation increased in the inflation-targeting period, and we find evidence of a unit root in several quantiles. In addition, the HLs increase in most of these quantiles.

Second, with respect to core inflation, similar to the period that preceded inflation-targeting (1998M7- 2006M12), the QKS test for the inflation-targeting period provides evidence of a unit root. This is in contrast with the evidence in favour of mean-reverting

core inflation that is found in the sub-sample just before the start of inflation-targeting (2007M1-2011M6). Although, the details of the persistence estimates for the specific quantiles during the inflation-targeting period are somewhat different from the earlier sub-samples, they retain the asymmetric property. Furthermore, the results reveal that the degree of persistence in core inflation is generally higher in most quantiles during the inflation-targeting period, especially at the 60%, 70% and 80% quantiles. The HL also increase significantly in all the quantiles during the inflation-targeting period. When considering these results, it is important to bear in mind that the inflation-targeting period includes periods when inflation rates in Uganda were at their highest, which could be attributed to large shocks that had very little to do with the implementation of the inflation-targeting regime (i.e. there is no counterfactual against which to measure the effect of this policy change).

In terms of macroeconomic management policy, the persistence results during the inflation-targeting period should be of concern to the central bank, as they suggest that the measure that the central bank targets for monetary policy implementation has become highly persistent over recent periods of time. These findings imply that higher inflation rates could be feeding higher inflation rate expectations. Therefore, the results should provoke discourse on the current measure of core inflation rate which the central bank targets, because they would lead to higher sacrifice ratios during periods of heightened inflationary pressure.

### 6.5 Wavelet decomposition

In this section, we consider an alternative measure of core inflation as the previous section established evidence of a unit root in the current measure of core inflation during the inflation-targeting period. [Figure 3](#) presents the original headline inflation rate (top panel), the smoothed stochastic trend (father wavelet) and the two corresponding wavelet details (mother wavelets). These wavelet functions may be interpreted in much the same way as the results from a Baxter and King (1999) or Christiano and Fitzgerald (2003), which provides estimates of the stochastic trend, cycle, and noise. In addition, given the nature of this data, the smoothed stochastic trend could be used as an estimate of the core inflation rate (Core-Wavelet). The wavelet details would then represent the cyclical features and noise in the rate of headline inflation.

After inspecting the results, it is worth noting that, when the inflation rate increases, the cycle plots widen and the level of noise increases. Similarly, when the inflation rate reduces, these two components contract. For example, when the inflation rate increased between 2011 and 2012, the noise in the headline inflation rate increased along with the cyclical component in inflation. From late 2012, when the inflation rate decelerated, the cyclical component decreased and the level of noise reduced. These results could suggest that the high frequency components in inflation were more volatile during periods of high inflation.

[Figure 4](#) plots the headline inflation rate (dotted line) and the Core-Wavelet inflation rate (solid line). It suggests that the two measures closely track each other (as should be the case), where the Core-Wavelet appears to be more persistent than the headline inflation rate.

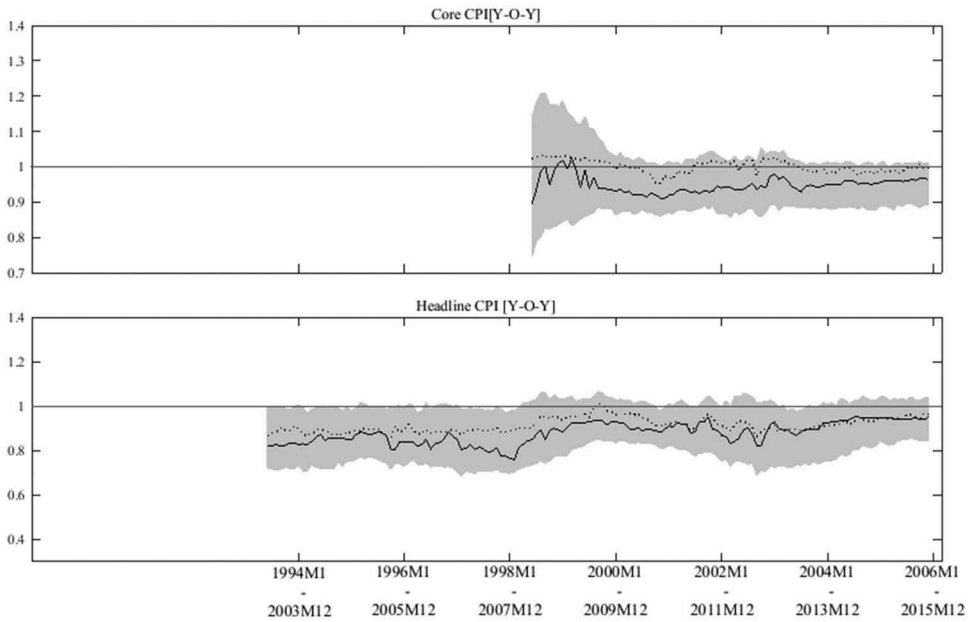


Figure 3. Daublet (4) wavelet decomposition of headline year-on-year inflation.

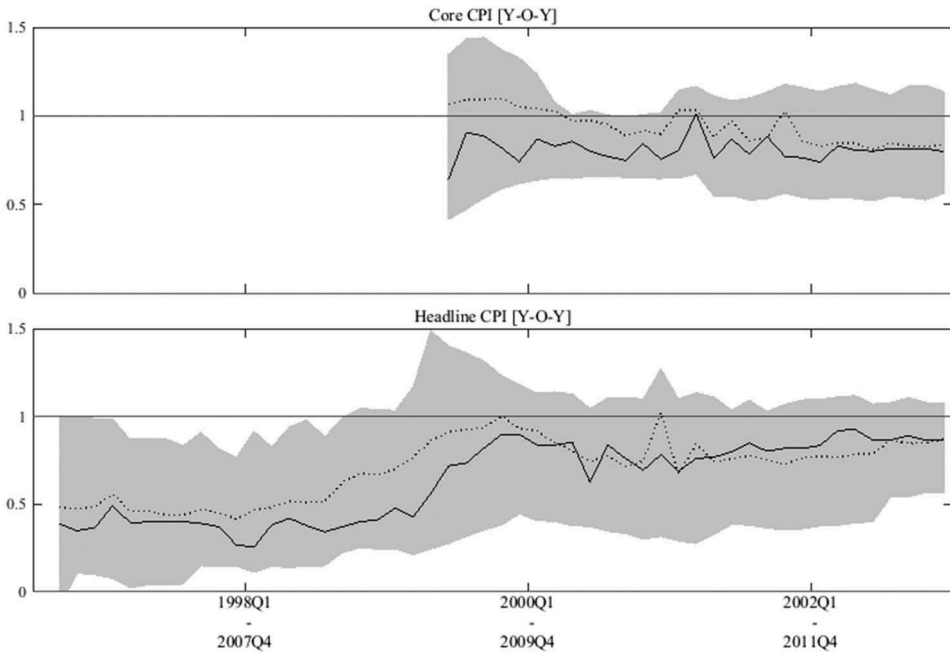
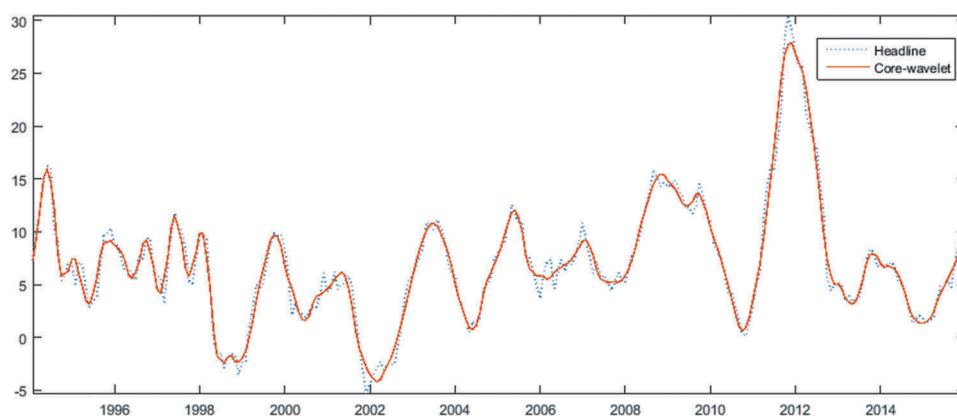


Figure 4. Headline inflation rate and core-wavelet inflation rate (year-on-year).

Figure 5 plots the current core inflation rate (Core-UBOS, represented by the solid line) and the Core-Wavelet (dotted line). The graph suggests that the Core-UBOS is more volatile than the Core-Wavelet. This could have important implications for monetary



**Figure 5.** Core-UBOS inflation rate and core-wavelet inflation rate (year-on-year).

policy formulation in Uganda. That is, a more volatile measure of core inflation might result in more frequent changes in monetary policy. This would in turn affect inflation expectations, as more frequent monetary policy changes may promote higher levels of uncertainty.

The above findings are supported by the results that are reported in [Table 5](#), which considers the volatility and correlation of the three measures of inflation. They confirm that the Core-UBOS measure is more volatile than the Core-Wavelet, and less correlated with both current and future headline inflation. Similarly, the Core-UBOS measure is also less correlated with the measure of core inflation that is obtained from the wavelet decomposition. The results in the table also suggest that the volatility of the wavelet measure is lower, while the measure is highly correlated with both current and future headline inflation. This suggests that the core wavelet based measure could be a useful target for monetary policy, as it could provide a more stable base on which to anchor inflation expectations.

### 6.6 Quantile regression results for core inflation obtained from wavelets

[Table 6](#) reports the results of the quantile regression unit root tests for the Core-Wavelet measure. These results are compared with the unit root test results from the Core-UBOS measure (extracted from [Table 4](#)). The results suggest that in sharp contrast with the Core-UBOS measure, we are able to reject the null of a unit root for the full sample (1993M7-2015M12) in the case of the Core-wavelet measure.

The detailed results for the specific quantiles during the inflation-targeting period, suggest that the estimates of  $\rho(\tau)$  that relate to the two measures are largely comparable. Although, the HLs in the Core-Wavelet measure are much higher, they do not reach the infinite level in the upper quantile. This implies that in the presence of large positive shocks, the alternative core measure would revert to its long-run equilibrium level, but only after a lengthy period of time. Moreover, similar to the Core-UBOS measure, persistence in the Core-Wavelet measure decreases sharply in the lowest quantile.

**Table 5.** Volatility and correlation.

	Standard Deviation	Correlations		
		Headline ( $t$ )	Headline ( $t + 1$ )	Core-UBOS
Headline	6.4028	1.0000	0.9664	0.9607
Core-UBOS	6.3516	0.9607	0.9240	1.0000
Core-Wavelet	6.3410	0.9889	0.9732	0.9576

$t$  refers to the current period,  $t + 1$  refers to future period.

**Table 6.** Quantile unit root tests for monthly Core-Wavelet inflation rate.

Variable	Period	$\tau$	0.20	0.30	0.40	0.50	0.60	0.70	0.80	
Core-Wavelet	1993M7-2015M12	$\hat{\rho}(\tau)$	0.99	1.00	0.89	1.00	1.00	1.00	0.99	
		Unit root	yes	yes	yes	yes	yes	yes	yes	
		$t$ -stat	-5.58	-0.12	0.29	1.05	0.42	0.02	-0.99	
		critical value	-2.42	-2.51	-2.60	-2.61	-2.51	-2.57	-2.57	
		Half-lives	60.30	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	
		KS-test	unit root: yes	QKS = 1.05		cv = 2.93				
Core-UBOS	1998M7-2015M12	$\hat{\rho}(\tau)$	0.96	0.97	0.97	0.99	0.98	0.98	0.98	
		Unit root	no	yes	yes	yes	yes	yes	yes	
		$t$ -stat	-5.01	-1.66	-1.65	-0.46	-0.81	-1.19	-2.58	
		critical value	-2.30	-2.34	-2.53	-2.44	-2.51	-2.31	-2.58	
		Half-lives	15.65	23.00	20.31	74.66	41.35	34.19	29.12	
		KS-test	unit root: no	QKS = 5.01		cv = 2.91				
Core-Wavelet	2011M7-2015M12	$\hat{\rho}(\tau)$	0.91	0.94	0.97	0.96	0.96	0.97	1.01	
		Unit root	no	yes	yes	yes	yes	yes	yes	
		$t$ -stat	-2.88	-1.80	-1.05	-1.11	-1.14	-0.70	0.27	
		critical value	-2.12	-2.28	-2.45	-2.30	-2.54	-2.30	-2.34	
		Half-lives	7.52	11.45	21.90	18.22	19.40	23.25	$\infty$	
		KS-test	unit root: yes	QKS = 2.88		cv = 2.91				

See notes in Table 4.

## 7 Conclusion

Price stability has become a key macroeconomic management objective for central banks in both advanced and developing countries. Hence, an investigation into the persistence of the inflationary process is important, particularly in those countries that have recently adopted inflation-targeting. This paper investigates the degree of persistence in Ugandan inflation by making use of a quantile regression approach that considers potential differences that may arise following shocks of different magnitudes. Moreover, this framework is more appropriate when applied to variables that exhibit heavy-tailed characteristics, as the preliminary results from the summary statistics suggest.

The results for the full sample suggest that the inflation rate in Uganda cannot be described as a unit root process, which implies that the effect of shocks dissipates as inflation returns to its long-run mean. However, we also find important differences in the adjustment process, as a unit root is found in most of the upper quantiles, suggesting

that large positive shocks are long-lived. When considering the inflation-targeting period, we find evidence of a unit root in the core rate of inflation, possibly suggesting that inflation expectations following the recent inflationary spiral could have become entrenched. This finding has important implications for monetary policy formulation in Uganda, as it suggests that the inflation measure that the central bank currently targets has recently become more persistent. This could result from the fact that inflation is now anchored at the target rate and any price shocks are going to be accompanied in many cases with a corresponding monetary policy shock, which will keep it very close to this rate for an extended period of time.

We then make use of wavelet transforms to construct an alternative measure of core inflation from headline inflation. We find that the alternative measure of core inflation is less volatile and more correlated with the headline inflation. This is important, since if monetary policy were to respond to such volatility, these actions could weaken the effectiveness of the inflation-targeting monetary policy framework (which acts as a tool for anchoring inflation expectations). Hence, the wavelets measure of core inflation could be considered as a useful alternative measure, which has provided similar results for the degree of persistence over the inflation-targeting period.

Lastly, to control inflation, policy-makers in the central bank would need to constantly monitor the build-up of inflation pressures, and pro-actively employ monetary policy tools to anchor inflation expectations, as the persistence in the measures of inflation are currently high.

## Notes

1. The reported measure of core inflation excludes volatile components such as energy and food prices.
2. The other two inflation measures energy, fuel and utility, and food crops only constitute about 17.6% of the overall inflation.
3. Stone (2003) inflation defines inflation-targeting lite as a monetary policy regime where central banks announce a broad inflation objective, however, as a result of their relatively low credibility, they may not be able to maintain an explicit rate of inflation over a period of time.
4. Half-lives measure the number of periods in which inflation rate remains above half of its initial level following a unit shock to the inflation rate.
5. Persistence is defined in the literature as the speed with which inflation converges to the equilibrium level, which makes it synonymous to the concept of an IRF for a  $AR(q)$  process.
6. By employing both univariate and multivariate reduced-form models, De Oliveira and Petrassi (2010), find a low and stable measure for inflation persistence in a group of 23 industrialized and 17 emerging economies. They note that the level of persistence for the developed countries is lower than in emerging economies.
7. The properties of these functions are described in Daubechies (1992).
8. In addition, concerns relating to the relatively high rate of inflation during this period of time could have anchored inflation expectations, and hence increased persistence.
9. Table 7 contains the results for the data that is measured at a quarterly frequency. It is contained in the appendix.
10. This period follows the realisation of the two inflationary spikes that were mentioned previously.

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## Disclosure statement

No potential conflict of interest was reported by the authors.

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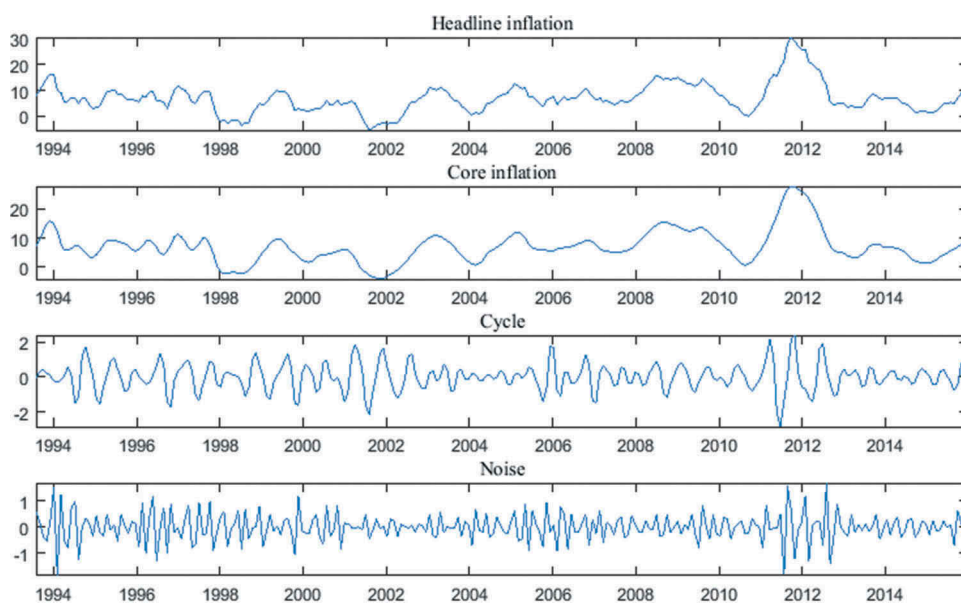
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## Appendix



**Figure A1.** Quarterly inflation rate persistence.

Notes: The graphs plot the estimates of  $\rho$  in the 10-year rolling window where the conditional mean and median are represented by the dotted and solid lines, respectively. The grey area represents the 95% bootstrap confidence intervals.

**Table A1.** Quantile unit root tests for quarterly inflation rate.

Variable	Period	$\tau$	0.20	0.30	0.40	0.50	0.60	0.70	0.80
Headline	1993Q3-2015Q4	$\hat{\rho}(\tau)$	0.63	0.67	0.68	0.67	0.67	0.69	0.75
		Unit root	no	no	no	no	no	no	yes
		$t$ -stat	-3.33	-3.25	-3.51	-3.67	-3.72	-2.79	-1.92
		critical value	-2.46	-2.56	-2.72	-2.58	-2.67	-2.72	-2.57
		Half-lives	1.50	1.73	1.81	1.73	1.73	1.84	2.47
		KS-test	unit root: no	QKS = 4.20 cv = 2.90					
	1993Q3-2006Q4	$\hat{\rho}(\tau)$	0.60	0.35	0.41	0.40	0.54	0.64	0.60
		Unit root	yes	yes	yes	no	yes	yes	yes
		$t$ -stat	-1.32	-2.16	-2.66	-3.14	-2.52	-2.21	-1.85
		critical value	-2.60	-2.68	-2.73	-2.70	-2.57	-2.59	-2.45
		Half-lives	1.36	0.67	0.79	0.77	1.11	1.54	1.35
		KS-test	unit root: no	QKS = 3.28 cv = 2.91					
	2007Q1-2015Q4	$\hat{\rho}(\tau)$	0.68	0.72	0.90	0.86	0.89	0.90	0.97
		Unit root	no	yes	yes	yes	yes	yes	yes
		$t$ -stat	-3.85	-1.95	-0.85	-1.23	-0.93	-1.80	-0.28
		critical value	-2.53	-2.66	-2.66	-2.64	-2.38	-2.53	-2.12
		Half-lives	1.82	0.67	0.79	0.77	1.11	1.54	1.35
		KS-test	unit root: no	QKS = 3.85 cv = 2.93					
Core	1998Q3-2015Q4	$\hat{\rho}(\tau)$	0.66	0.64	0.79	0.79	0.90	0.92	1.05
		Unit root	no	no	no	no	yes	yes	yes
		$t$ -stat	-3.84	-5.35	-3.34	-2.81	-1.45	-1.00	0.44
		critical value	-2.20	-2.44	-2.40	-2.43	-2.56	-2.52	-2.39
		Half-lives	1.69	1.55	3.01	2.89	6.36	8.08	$\infty$
		KS-test	unit root: no	QKS = 5.35 cv = 2.91					
	1993Q3-2006Q4	$\hat{\rho}(\tau)$	0.24	0.35	0.41	0.46	0.53	0.43	0.71
		Unit root	yes	yes	yes	yes	yes	no	yes
		$t$ -stat	-2.15	-1.77	-1.51	-2.09	-2.10	-2.55	-1.52
		critical value	-2.70	-2.80	-2.76	-2.55	-2.41	-2.15	-2.12
		Half-lives	0.48	0.66	0.77	0.88	1.09	0.81	2.02
		KS-test	unit root: yes	QKS = 2.55 cv = 2.87					
	2007Q1-2015Q4	$\hat{\rho}(\tau)$	0.61	0.61	0.74	0.81	0.88	1.03	1.07
		Unit root	no	no	yes	yes	yes	yes	yes
		$t$ -stat	-4.36	-4.85	-2.26	-1.61	-0.73	0.15	0.45
		critical value	-2.61	-2.54	-2.47	-2.57	-2.70	-2.56	-2.77
		Half-lives	1.69	1.55	3.01	2.89	6.36	$\infty$	$\infty$
		KS-test	unit root: no	QKS = 4.85 cv = 2.91					

Critical value (cv) at the 5% significance level. QKS statistics are calculated for the entire sample. We reject the null when the calculated test statistics is less than the critical value. Infinity HLs mean  $\hat{\rho}(\tau)$  is larger than unity.