Available online at **globets.org/journal**
International Journal of Education, Technology and Science
5(2) (2025) 160–184

IJETS
International Journal of Education Technology and Science

CONCEPTUAL AND PROCEDURAL TRAJECTORIES IN TRANSFORMATION GEOMETRY: A COMPARATIVE STUDY OF TECHNOLOGY-ENHANCED AND CONVENTIONAL VAN HIELE PHASED INSTRUCTION

(Research Article)

Issa Ndungo[a1], Sudi Balimuttajjo[b], Edwin Akugizibwe[c]

[a,b] *Mbarara University of Science and Technology; Faculty of Science, Uganda*
[a, c] *Mountains of the Moon University, Faculty of Science, Technology and Innovations, Uganda*

Received: 25.01.2025 Revised version received: 16.04.2025 Accepted: 20.04.2025

Abstract

This study investigates the effectiveness of Technology-Enhanced Van Hiele Phased Instruction (TVHPI) compared to Conventional Van Hiele Phased Instruction (CVHPI) in enhancing learners' conceptual understanding and procedural fluency in transformation geometry. A quasi-experimental design was employed, involving 144 secondary school learners from six schools in Uganda. Conceptual and procedural understanding was assessed from learners' test scripts using a 5-point performance scale, and performance across Van Hiele levels was analyzed using descriptive and inferential statistics. Results revealed that TVHPI significantly improved both conceptual understanding ($t = 10.861, p = 0.00t = 10.861, p = 0.00$) and procedural fluency (compared to CVHPI. Performance across Van Hiele levels showed foundational tasks (Levels 1 and 2) were better supported by both strategies, while TVHPI showed an advantage at intermediate levels (Level 3). The study recommends integrating technology like GeoGebra into instructional practices to enhance learning outcomes. These findings emphasize the potential of technology-enhanced strategies to improve geometric reasoning and inform curriculum design.

Keywords: Technology-enhanced learning, Van Hiele Phased instruction, conceptual understanding, transformation geometry

© 2021 IJETS. Published by *International Journal of Education Technology and Science (IJETS*). Copyright for this article is retained by the author(s), with first publication rights granted to the Journal. This is an open-access article distributed under the terms and conditions of the Creative Commons Attribution license (CC BY-NC-ND)(**http://creativecommons.org/licenses/by-nc-nd/4.0/**).

---

[1] Corresponding author : Issa Ndungoa. ORCID ID.: https://orcid.org/0000-0003-0492-764x
E-mail: ndungoissa@yahoo.com

1. Introduction

1.1 Background

Mathematics education has long grappled with balancing conceptual understanding and procedural fluency to produce well-rounded learners capable of applying mathematical knowledge in diverse contexts. Globally, educational systems have adopted varied instructional strategies to achieve this balance, yet disparities in students' outcomes persist (Bartell et al., 2013; Chappell & Killpatrick, 2003; Crooks & Alibali, 2014; Edulsa, 2022). The arrival of technology in education has introduced innovative methods such as Technology-Enhanced Van Hiele Phased Instruction (TVHPI), offering new possibilities for addressing these challenges (Alkhateeb & Al-Duwairi, 2019; Bekene, 2020; Mosese & Ogbonnaya, 2021; Ngwabe & Felix, 2020; Zengin, 2017). This study investigates how TVHPI compares to Conventional Van Hiele Phased Instruction (CVHPI) in enhancing students' conceptual and procedural understanding, contributing to the broader discourse on effective mathematics instruction.

Globally, the focus on improving students' mathematical proficiency has emphasized the need to integrate technology into instructional practices (Ndungo et al., 2025). Studies have highlighted significant deficiencies in conceptual understanding among students, particularly in foundational topics like fractions, algebra, and geometry (Richland et al., 2012; Salim & Gilar, 2020). These deficiencies often result from overemphasizing procedural fluency at the expense of deeper comprehension. Technology-enhanced methods, such as using GeoGebra in TVHPI, aim to bridge this gap by providing dynamic visualization tools that foster relational understanding, thereby addressing global calls for instructional innovation in mathematics education.

This study is grounded in the Van Hiele Theory of Geometric reasoning, which posits that learners progress through hierarchical levels of understanding in geometry. The theory emphasizes phased instruction, guiding learners from visualization to rigorous abstraction (Crowley, 1987; Machisi & Feza, 2021; Moru et al., 2021; Pujawan et al., 2020; Usiskin, 1982). Traditional approaches to Van Hiele Phased Instruction (VHPI) rely heavily on static methods, such as drawing and graph paper, which may limit students' capacity to visualize complex relationships. In contrast, TVHPI incorporates dynamic tools, aligning with constructivist theories that advocate for active learner engagement through exploration and interaction with digital representations. By comparing TVHPI and CVHPI, this study contributes to understanding how technological enhancements align with theoretical principles to improve learning outcomes.

The Conventional Van Hiele Phased Instruction (CVHPI) is a structured, teacher-led approach grounded in Van Hiele's theory of geometric reasoning. It utilizes traditional tools such as graph paper, mirrors, and compasses to guide learners through the phases of geometric understanding. Emphasis is placed on step-by-step instruction, deliberate practice, and reinforcement through routine exercises. Feedback is typically provided after task completion, making the learning process more sequential and dependent on teacher evaluation. In contrast, the Technology-Enhanced Van Hiele Phased Instruction (TVHPI) integrates the same theoretical framework with dynamic digital tools like GeoGebra. This approach allows learners to interact with and manipulate geometric concepts visually and in real-time, fostering active engagement and exploration (Geiger et al., 2012; Sunzuma, 2023). TVHPI provides immediate feedback, enabling learners to correct errors and refine their understanding. The key distinction between TVHPI and CVHPI lies in their mode of delivery. TVHPI leverages technology to create a dynamic, feedback-rich learning environment, empowering learners to engage with geometric concepts actively. On the other hand, CVHPI emphasizes systematic, teacher-led instruction using static materials, offering a more traditional, structured learning experience.

The conceptual foundation of this study lies in the interplay between conceptual and procedural understanding. Conceptual understanding refers to students' ability to comprehend mathematical ideas and see relationships between concepts, while procedural fluency involves executing mathematical procedures accurately and efficiently (Bossé & Bahr, 2008; Richland et al., 2012). Technology integration in TVHPI seeks to balance these dimensions by enabling learners to experiment with transformations dynamically, fostering connections between geometric concepts and their procedural applications. This dual focus aligns with research advocating integrating conceptual and procedural knowledge to enhance mathematical proficiency (Kasmer & Kim, 2011; Salim & Gilar, 2020).

Like many others in Sub-Saharan Africa, the Ugandan education system faces challenges in delivering effective mathematics instruction (Ndungo et al., 2024). Conventional methods often rely on rote memorization, with limited integration of technology or active learning strategies. As a result, students struggle with both conceptual and procedural aspects of mathematics, particularly in geometry. Previous studies in this context have demonstrated the potential of interventions like TVHPI to enhance understanding by leveraging dynamic tools such as GeoGebra (Ngwabe & Felix, 2020; Praveen & Leong, 2013; Zengin, 2017). This study compares TVHPI to CVHPI in Ugandan classrooms, aiming to identify how these approaches impact students' abilities to grasp and apply geometric transformations. The findings aim to contribute insights applicable to similar educational contexts globally by addressing these localized challenges.

Therefore, this paper seeks to answer the question: How does TVHPI compare to CVHPI in enhancing students' conceptual and procedural understanding? By examining the comparative efficacy of these instructional strategies, the study aspires to inform policy and practice, providing a framework for integrating technology to achieve balanced and effective mathematics instruction.

1.2 Problem Statement

Mathematics education, particularly in geometry, continues to face challenges in fostering deep conceptual and procedural understanding among learners. Traditional instructional methods often focus on memorization of formulas and rote application of procedures, resulting in superficial learning and limited ability to solve complex problems (Lestari & Surya, 2017). These challenges are especially pronounced in transformation geometry, as students struggle with visualizing geometric relationships, understanding underlying concepts, and applying procedures effectively (Ndungo et al., 2024).

The Van Hiele model of geometric reasoning provides a structured framework for advancing learners through levels of understanding. However, its potential is often underutilized in classrooms, where the absence of interactive tools limits its effectiveness. Emerging technologies, such as GeoGebra, offer opportunities to enhance the Van Hiele Phased Instruction (VHPI) model by providing dynamic visualization, interactive exploration, and practical applications of geometric concepts (Bekene Bedada & Machaba, 2022; Khalil et al., 2019; Mollakuqe et al., 2020; Mukamba & Makamure, 2020; Narh-kert & Sabtiwu, 2022).

Despite the promising capabilities of technology-enhanced instruction, limited research has evaluated its impact on conceptual understanding (the ability to grasp mathematical ideas and relationships) and procedural understanding (the application of mathematical processes). Furthermore, there is insufficient evidence on how integrating technology into the Van Hiele framework can address common barriers in geometry education, such as low engagement, difficulty in visualizing concepts, and procedural errors.

This study seeks to fill this gap by examining the effectiveness of Technology-Enhanced Van Hiele Phased Instruction (TVHPI) in developing students' conceptual and procedural understanding of transformation geometry. The findings aim to provide actionable insights for educators and policymakers seeking to improve instructional strategies and leverage technology to enhance learning outcomes in mathematics.

1.3 Objective of the Study

This study aims to compare the effectiveness of TVHPI and CVHPI in enhancing learners’ conceptual understanding and procedural fluency, analyzing performance across Van Hiele levels of geometric reasoning.

1.4 Research Questions

1. How does TVHPI influence learners’ conceptual understanding compared to CVHPI?
2. How does TVHPI affect learners’ procedural fluency compared to CVHPI?
3. At which Van Hiele levels do learners perform better in conceptual and procedural tasks under TVHPI and CVHPI?

1.5 Significance of the Study

This study provides insights into the effectiveness of integrating technology in geometry instruction, offering evidence to guide teaching practices, curriculum development, and policy decisions. Analyzing learners' performance across Van Hiele levels highlights areas where interventions can support improved geometric reasoning. The findings contribute to the global discourse on modernizing mathematics education, particularly in Uganda's Competency-Based Curriculum (CBC) context.

1.6 Scope of the Study

The study focuses on transformation geometry, including reflection, rotation, enlargement, and translation, using the Van Hiele framework to analyze reasoning stages. It involves 483 learners from six schools in Midwestern Uganda, with 144 purposively selected for detailed analysis. Using traditional methods, learners were taught using either TVHPI with tools like GeoGebra or CVHPI. Performance was assessed through a test with 20 questions, categorized into conceptual and procedural tasks, to evaluate the impact of each strategy on learners' geometric reasoning.

2. Literature Review

Historically, research has consistently highlighted challenges in balancing conceptual understanding and procedural knowledge. Eisenhart et al. (1993) emphasized that novice mathematics teachers often prioritize procedural fluency due to systemic constraints and curriculum demands. This procedural dominance persisted despite teachers' intentions to teach for understanding, illustrating the gap between theory and classroom practice. Building on this, Chappell and Killpatrick (2003) examined the effectiveness of concept-based instruction in calculus education, demonstrating that conceptual learning could be enhanced without sacrificing procedural rigor. These findings challenged earlier assumptions that focusing on one form of knowledge might detract from the other.

More recently, Bahr and Bossé (2008) extended these discussions to teacher education, highlighting the diverse views among educators on balancing conceptual and procedural knowledge. Their study advocated integrating both knowledge forms to develop flexible mathematical thinking. This aligns with Star (2020), who argued for "procedural understanding" as a deeper form of procedural knowledge, highlighting the interdependence of the two.

Early interventions often relied on teacher-centered approaches. For example, Lestari and Surya (2017) demonstrated that Realistic Mathematics Education (RME) significantly improved conceptual understanding compared to traditional lecture methods. Similarly, Ploger and Hecht (2009) showed that using Chartworld software enhanced students' conceptual and procedural knowledge of number operations, particularly through exploratory learning.

Building on these findings, Asfar and Asfar (2020) introduced Case-Based Games Learning (CBGL) with Quizizz, revealing higher engagement and improved understanding compared to traditional case-based methods. These innovations highlight a gradual shift toward interactive, technology-enhanced strategies to address conceptual deficits. However, these studies often focused on single topics, such as Pythagoras' theorem or number operations, limiting their generalizability.

The importance of visualization and semiotic tools in mathematics education has been widely recognized. Mudaly (2014) emphasized the role of semiotic mediation in helping learners understand graphical functional relationships, addressing their reliance on visual memorization rather than mathematical reasoning. Malatjie and Machaba (2019) further explored concept mapping to enhance understanding of transformation geometry, revealing its potential to link concepts and foster relational thinking.

Despite these advances, both studies were limited in scope, with small sample sizes and narrow geographical focus. Their findings suggest the need for broader research that integrates visual tools with other instructional strategies to address diverse mathematical challenges.

Kanive et al. (2014) highlighted the benefits of computer-based practice in improving retention and computational fluency among struggling learners. However, the study found limited impact on conceptual understanding, reflecting a common critique of technology-driven interventions. In contrast, Richland et al. (2012) argued that relational thinking and representation comparisons, often facilitated through digital tools, were essential for deeper learning.

More recent studies, such as those by Mendezabal and Tindowen (2018), demonstrated the potential of Microsoft Mathematics to improve attitudes and skills in Differential Calculus. While these tools foster engagement, their long-term impact on conceptual understanding remains underexplored, especially in comparative contexts.

Kusumaningsih et al. (2019) examined gender differences in conceptual understanding, finding that male students outperformed their female peers in certain aspects, although both genders displayed similar abilities in applying learned concepts. These findings align with Salim and Gilar (2020), who highlighted the interdependence of conceptual and procedural knowledge in solving fraction problems. Both studies emphasize the need for tailored interventions for individual cognitive styles and learning needs.

Several studies have emphasized the interplay between conceptual and procedural knowledge. Salim and Gilar (2020) showed that balanced competencies significantly improved problem-solving abilities in fractions. Similarly, Chappell and Killpatrick (2003) found that concept-based instruction enhanced understanding without compromising procedural skills. These findings align with Star's (2000) argument that deeper procedural understanding complements conceptual knowledge, providing a holistic foundation for mathematical proficiency. However, the broader literature reveals a gap in comparative studies that explicitly evaluate instructional strategies for balancing these competencies. Most studies either focus on one form of knowledge or examine their relationship in isolation, leaving unanswered questions about how best to integrate them.

This study addresses these gaps by investigating how Technology-Enhanced Van Hiele Phased Instruction (TVHPI) compares to Conventional Van Hiele Phased Instruction (CVHPI) in enhancing conceptual and procedural understanding. TVHPI offers an interactive, dynamic learning environment that aligns with theoretical and empirical insights by focusing on geometric transformations and integrating tools like GeoGebra. This approach bridges global and contextual gaps and provides practical solutions for improving mathematics instruction in diverse educational settings.

3. Methodology

3.1 Philosophical Underpinning

This study is grounded in the positivist paradigm, which emphasizes objective observation and measurement of phenomena to establish cause-and-effect relationships. The positivist approach aligns with the study's goal of quantitatively evaluating the effectiveness of Technology-Enhanced Van Hiele Phased Instruction (TVHPI) and Conventional Van Hiele Phased Instruction (CVHPI). This study focuses on measurable outcomes and provides evidence-based insights into learners’ conceptual understanding and procedural fluency in geometric reasoning.

3.2 Research Design

The study employed a quasi-experimental design to compare the effectiveness of TVHPI and CVHPI. This design was chosen to allow controlled comparisons between the two instructional strategies within natural classroom settings, avoiding the need for artificial manipulation. The quasi-experimental approach also ensured that data collection reflected real-world teaching practices while maintaining a robust framework for assessing the impact of the interventions.

3.3 Participants

Four hundred eighty-three secondary school learners of S.3 (aged 16 to 17 years) from six schools in mid-western Uganda participated in the study. Both the TVHPI and CVHPI instructional strategies were implemented in each school during the intervention. For detailed analysis, 144 learners were purposively selected from the larger cohort. The primary criterion for selection was ensuring relative equality in baseline performance (pretest scores) between the TVHPI and CVHPI groups. Additional considerations, such as gender, location (urban and rural), and Van Hiele levels of geometric reasoning, were addressed to provide a representative and comparable sample.

3.4 Sampling and Sample Size

The purposive sampling process involved selecting 24 learners from each school, divided equally between the TVHPI and CVHPI groups. The selection focused on achieving comparable baseline scores across groups to mitigate pre-existing differences. Once the baseline balance was ensured, other factors, such as gender (six males and six females in each group) and location (equal representation from urban and rural areas), were considered. This sampling approach provided a balanced dataset for analyzing instructional effects while accounting for demographic diversity and ensuring valid comparisons.

3.5 Data Collection Tools

The primary data collection tool was a geometry test designed to assess learners' conceptual understanding, procedural fluency, and the integration of these skills in dual-skill tasks. Conceptual understanding was assessed through test items to evaluate learners' ability to explain, identify, or infer geometric relationships. Procedural fluency was measured using tasks requiring learners to execute specific geometric transformations or solve problems step-by-step. Both constructs were evaluated using a scale of 1 to 5, where 1 indicated low performance and 5 represented excellent performance.

A total of 20 questions were distributed evenly across the first four Van Hiele levels, with five questions at each level (the last level was not tested due to the curriculum requirement of the participating class(Ndungo, 2024)). Conceptual questions (40%) assessed reasoning, understanding of geometric principles, and relationships between properties, such as identifying transformations and explaining rotational symmetry. Procedural questions (35%) focused on executing tasks like performing transformations, calculating transformation matrices, or plotting geometric figures. Dual-skill questions (25%) combined conceptual reasoning and procedural execution, requiring learners to apply both skills to solve more complex tasks, such as determining the area scale factor during dilation or integrating multiple transformations to optimize a path. For the analysis, dual-skill questions were categorized into separate conceptual and procedural components, allowing for precise performance comparisons across instructional strategies.

At Level 1 (Visualization), questions focused on recognizing geometric transformations and symmetry. For example, learners were asked to name the type of transformation or reflect points across axes. Level 2 (Analysis) emphasized identifying relationships between geometric properties and performing simple transformations, such as calculating scale factors or executing consecutive reflections. Level 3 (Abstraction) transitioned toward integrating reasoning and procedural skills, with tasks like dilating objects, calculating areas, or interpreting transformation matrices. Finally, Level 4 (Deduction) required learners to use formal reasoning, combining transformations like rotation, enlargement, and translation to solve complex real-world scenarios. The categorization ensured a balanced assessment of learners' abilities across all levels and skill types.

3.6 Procedure

The intervention began with teacher training tailored to the two instructional methods. After training, teachers implemented either the TVHPI or CVHPI strategies in their classrooms. Learners in the TVHPI group utilized the GeoGebra dynamic tool to explore geometric concepts interactively. In contrast, those in the CVHPI group relied on traditional approaches, including graph paper and physical manipulatives. The teacher training followed the Van Hiele phased instruction and the use of GeoGebra. Before instruction, all 483 learners were tested (pre-test), and their results were used to select 144 learners to participate in the study based on demographic balance and comparable performance profiles derived from their pretest results, ensuring equivalence in baseline understanding. After six weeks of intervention, learners were subjected to a post-test to measure their conceptual and procedural understanding. This test formed the primary data informing tool for this study. The training protocol of the study is accessible at: https://www.academia.edu/122616131/Teacher_Training_Manual_Issa_Ndungo.

3.7 Data Analysis

The data analysis was structured to address each research question quantitatively. Parametric tests were employed to analyze differences in conceptual understanding and procedural fluency (Research Questions 1 and 2), as the differences in mean scores between the TVHPI and CVHPI groups were normally distributed, as confirmed by the Shapiro-Wilk test. For Research Question 3, non-parametric tests (Kruskal-Wallis) were used due to the ordinal nature of Van Hiele levels, which reflect a hierarchical progression in geometric reasoning. Moreover, descriptive statistics were calculated for mean and standard deviation scores in all three research questions.

4. Results

The results presented in this section address the three research questions, providing insights into how the TVHPI compares to the CVHPI in fostering learners' conceptual understanding and procedural fluency and examining performance variations across Van Hiele levels of geometric reasoning. The subsection starts by presenting results on the impact of TVHPI on learners' conceptual understanding compared to CVHPI, followed by the impact of TVHPI on learners' procedural fluency understanding compared to CVHPI. It ends with performance across Van Hiele levels under TVHPI and CVHPI.

4.1 Impact of TVHPI on Learners' Conceptual Understanding Compared to CVHPI

This subsection tests whether the TVHPI strategy leads to a significantly better conceptual understanding of geometric concepts than the CVHPI strategy. First, mean values of conceptual understanding scores between the two groups of learners are compared in Table 1, followed by the results of a two-sample independent t-test to determine whether the difference in mean conceptual understanding scores between the TVHPI and CVHPI groups is significant.

Table 1: Descriptive Statistics for conceptual scores per strategy

Instructional Strategy	**Count**	**Mean**	**Std Dev**	**Min**	**25th Percentile**	**Median (50%)**	**75th Percentile**	**Max**
CVHPI	72	2.26	0.38	1.51	1.99	2.18	2.47	3.22
TVHPI	72	2.90	0.32	2.07	2.73	2.90	3.12	3.66

Descriptive statistical analyses were computed to summarize learners' conceptual understanding scores under the two instructional strategies: Conventional Van Hiele Phased Instruction (CVHPI) and Technology-Enhanced Van Hiele Phased Instruction (TVHPI). Each instructional group included 72 learners. The mean conceptual score for the CVHPI group was 2.26 (SD = 0.38), whereas learners in the TVHPI group attained a higher mean score of 2.90 (SD = 0.32). This difference reflects a substantial shift in average performance favoring the technology-enhanced approach. Additionally, the score range for TVHPI (Min = 2.07, Max = 3.66) was more elevated overall than CVHPI (Min = 1.51, Max = 3.22). The median values further highlight this performance gap. The median for TVHPI was 2.90, while the median for CVHPI was 2.18. The 25th and 75th percentiles for the TVHPI group (2.73 and 3.12, respectively) were also higher than those for the CVHPI group (1.99 and 2.47, respectively), indicating that even the lowest-performing learners under TVHPI scored above the mid-range of those taught with CVHPI.

Therefore, the descriptive data support the conclusion that TVHPI significantly enhances learners' conceptual understanding in geometry. Not only are the scores consistently higher across the board, but the narrower spread and elevated minimum scores also suggest improved equity in learning outcomes, likely due to the visual and interactive affordances of technology-integrated instruction.

To statistically conclude on the comparison of TVHPI and CVHPI, evaluate the impact of instructional strategy on learners' conceptual understanding of transformation geometry, an independent samples t-test was conducted to compare the mean scores between the Technology-Enhanced Van Hiele Phased Instruction (TVHPI) group and the Conventional Van Hiele Phased Instruction (CVHPI) group. The results revealed a statistically significant difference between the two groups. The p-value, being well below the conventional alpha level of 0.05, indicates that learners exposed to the TVHPI strategy demonstrated significantly higher levels of conceptual understanding than their counterparts in the CVHPI group.

Furthermore, the higher mean scores and reduced variability observed in the TVHPI group suggest that technology-enhanced instruction effectively scaffolded learners' development of a deeper conceptual grasp of geometric relationships, particularly integrating dynamic visualization tools such as GeoGebra. These findings support the argument that technology can enrich the Van Hiele instructional phases by making abstract geometric concepts more accessible and cognitively engaging.

The findings prove that TVHPI is more effective than CVHPI in enhancing conceptual understanding. They confirm that the TVHPI strategy significantly improves learners' conceptual understanding of geometry compared to the CVHPI strategy. These results emphasize the potential of technology-enhanced methods to foster deeper comprehension of geometric concepts and improve learning outcomes.

4.2 Impact of TVHPI on Learners' Procedural Fluency Compared to CVHPI

The results of this second question examine whether learners exposed to TVHPI achieve significantly higher procedural fluency than those taught using CVHPI. Table 2 summarizes the descriptive statistics for procedural fluency scores under TVHPI and CVHPI, followed by a presentation of the independent t-test results that determine if the difference in procedural fluency between the two instructional strategies is statistically significant.

Table 2: Descriptive Statistics for Procedural Fluency

Instructional Strategy	**Count**	**Mean**	**Std. Dev.**	**Min**	**25%**	**Median**	**75%**	**Max**
CVHPI	72	2.44	0.40	1.65	2.14	2.37	2.74	3.46
TVHPI	72	3.06	0.31	2.37	2.82	3.08	3.31	3.81

Descriptive statistics were calculated to examine the distribution of scores in procedural fluency across the two instructional strategies: Conventional Van Hiele Phased Instruction (CVHPI) and Technology-Enhanced Van Hiele Phased Instruction (TVHPI). Each group consisted of 72 learners. The results revealed a marked difference in overall performance.

Learners in the TVHPI group achieved a higher mean score (M = 3.06, SD = 0.31) than their CVHPI group counterparts (M = 2.44, SD = 0.40). Additionally, the range of scores was narrower for the TVHPI group (Min = 2.37, Max = 3.81) than for the CVHPI group (Min = 1.65, Max = 3.46), suggesting greater consistency in performance.

Quartile values further highlighted this difference. The median score for TVHPI was 3.08, with 25th and 75th percentiles at 2.82 and 3.31, respectively. In contrast, the CVHPI group had a lower median of 2.37, with the 25th percentile at 2.14 and the 75th percentile at 2.74. These results indicate that learners in the TVHPI group not only performed better on average, but the distribution of scores was more tightly clustered around the higher end, reflecting reduced variability and a more uniform improvement. These descriptive findings support the inferential results that TVHPI is more effective in promoting procedural fluency in geometry. Consistent learner performance also suggests that technology-enhanced instruction may provide scaffolding that supports diverse learners more equitably, narrowing performance gaps.

An independent samples t-test was conducted to determine whether the observed difference in procedural fluency between the TVHPI and CVHPI instructional groups was statistically significant. The results revealed a substantial and statistically significant difference, $t(142) = 10.491, p < .001$$t(142) = 10.491, p < .001$. The p-value, well below the conventional threshold of 0.05, confirms that learners in the TVHPI group demonstrated significantly higher procedural fluency than those in the CVHPI group. This finding highlights the advantage of integrating technology into geometry instruction. In particular, dynamic digital tools, such as GeoGebra, enhance learners' ability to execute geometric transformations accurately and efficiently. Unlike the CVHPI approach, which relies heavily on static tools such as graph paper and physical manipulatives, the TVHPI strategy offers interactive features, immediate feedback, and visual representations that facilitate deeper engagement with procedural processes. Moreover, the observed consistency in performance within the TVHPI group, as evidenced by a lower standard deviation, suggests that technology-enhanced instruction improves average performance and helps bridge gaps among learners. This more uniform distribution of scores implies that such tools may support differentiated learning by enabling students at varying proficiency levels to progress more evenly in mastering procedural tasks.

4.3 Comparing Conceptual and Procedural Achievement across TVHPI and CVHPI

A comparison of conceptual and procedural understanding across the two instructional strategies, TVHPI and CVHPI, reveals converging and diverging learner outcomes patterns. Learners under the TVHPI strategy consistently outperformed those in the CVHPI group in both conceptual and procedural tasks, with higher mean scores and reduced variability. However, the magnitude and nature of improvement varied between the two skill domains. In conceptual understanding, the difference was not only statistically significant but also structurally distinct; learners in the TVHPI group exhibited elevated scores across all percentiles, suggesting that technology played a pivotal role in enhancing deep reasoning and comprehension of geometric principles. By contrast, while procedural fluency also improved significantly under TVHPI, the distribution of scores was slightly more spread, indicating that gains in execution of tasks were present but potentially influenced by learners' prior familiarity with digital tools or individual practice habits. Interestingly, the CVHPI group demonstrated relatively better consistency in procedural routines than conceptual reasoning, likely due to the emphasis on traditional repetition and static visualization techniques such as graph paper and tracing. These findings suggest that while both domains benefit from technology integration, conceptual understanding appears to be more sensitive to the affordances of interactive tools. In contrast, procedural fluency can be moderately supported through either approach, though more robustly under TVHPI. Figure 1 shows the learning trajectories to illustrate this comparison further.

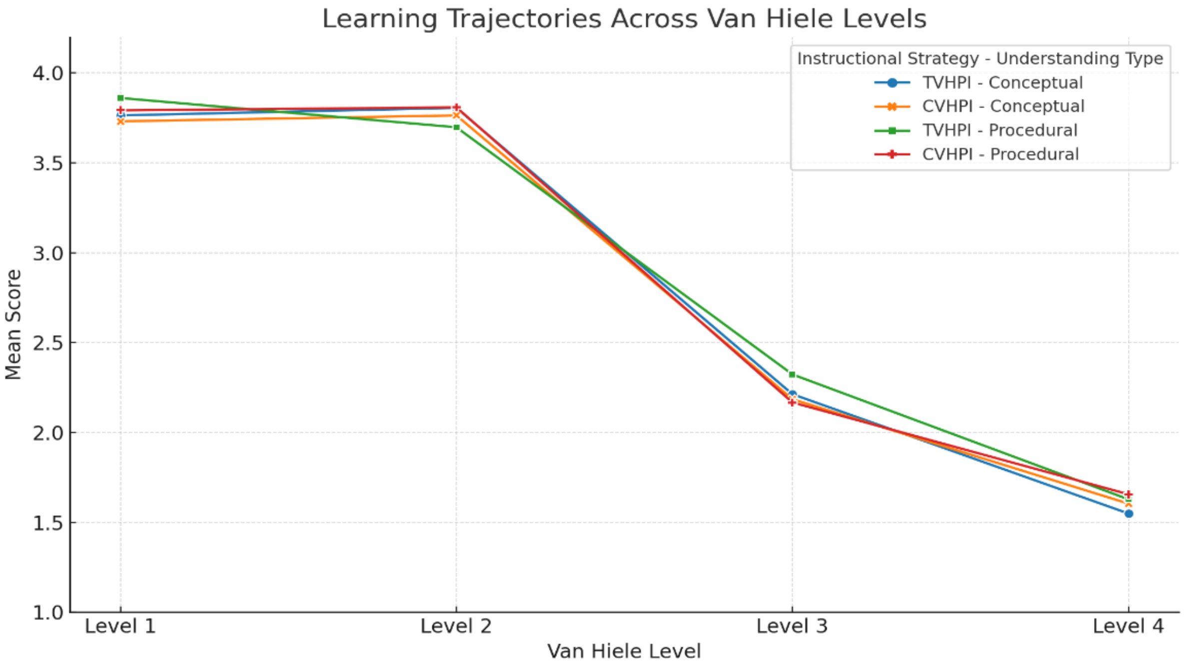

Figure 1: Learning Trajectory across Van Hiele Levels

Figure 1 illustrates the learning trajectories of learners' conceptual and procedural understanding across the four Van Hiele levels under two instructional strategies: Technology-Enhanced Van Hiele Phased Instruction (TVHPI) and Conventional Van Hiele Phased Instruction (CVHPI). The line graph reveals that learners in the TVHPI group consistently achieved higher mean scores than those in the CVHPI group across all levels and understanding types. Notably, both groups experienced a gradual decline in performance from Level 1 to Level 4, aligning with the increasing cognitive complexity defined by the Van Hiele model. However, the decline was more moderate in the TVHPI group, suggesting that integrating technology (e.g., GeoGebra) may provide better scaffolding for deeper reasoning and procedural execution at higher levels. These patterns reflect the added instructional value of technology in supporting progression through geometric reasoning stages. Figure 2 demonstrates the difference in means for conceptual and procedural understanding for the two instructional strategies at different Van Hiele levels.

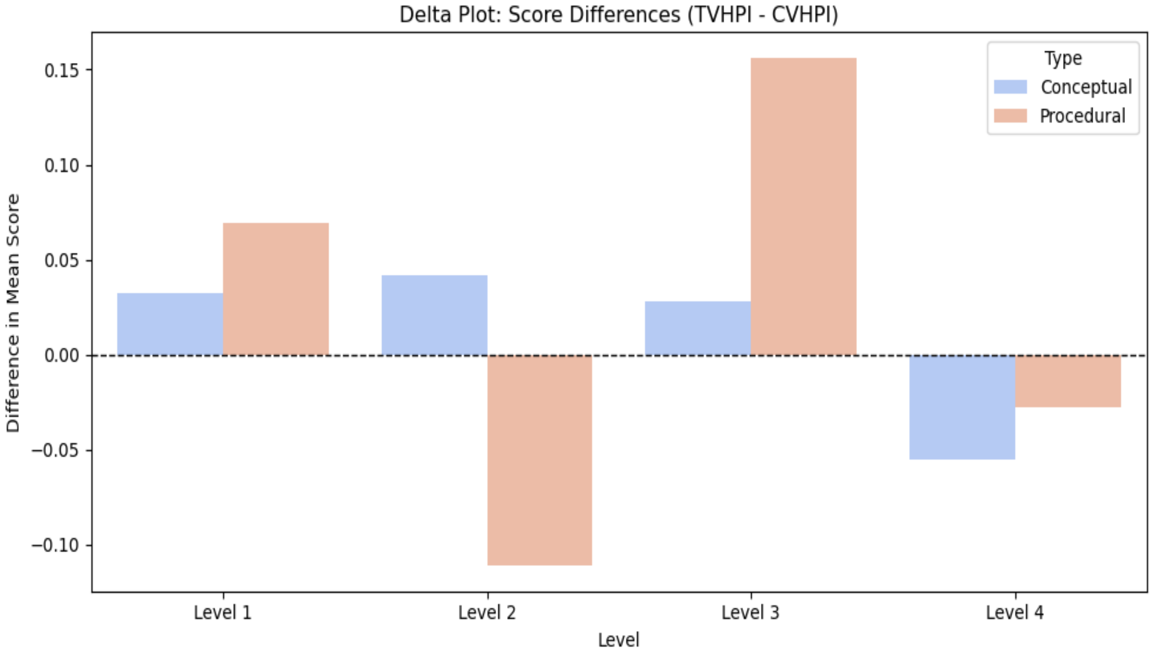

Figure 2: Delta Plot: Score Differences (TVHPI-CVHPI)

Figure 2 presents a delta plot that visualizes the mean score differences between learners taught using TVHPI and those taught using CVHPI across Van Hiele levels and understanding types. Positive bars indicate areas where TVHPI outperformed CVHPI, while negative values represent the opposite. The plot shows consistent positive differences across nearly all levels, with the most notable gains in Level 3 procedural and Level 2 conceptual understanding. These findings suggest that TVHPI not only enhances basic comprehension and skill execution but also contributes significantly to learners' performance in tasks that require integration of prior knowledge and higher-order reasoning. The plot provides a succinct representation of the instructional advantage of TVHPI and highlights areas where this strategy had the most substantial pedagogical impact.

4.4 Performance across Van Hiele Levels under TVHPI and CVHPI

The goal here is to identify which instructional strategy, TVHPI or CVHPI, better supports learners at various levels of geometric complexity. The analysis focuses on conceptual, procedural, and combined understanding to identify trends and differences at each level. Descriptive statistics, Performance across Van Hiele Levels, were conducted to evaluate performance trends and differences across these levels. The results are shown in Table 3. Additionally, statistical comparison across Van Hiele levels was conducted to determine the observed differences in performance across levels.

Table 3: Descriptive Statistics across Van Hiele Levels

Van Hiele Level	Instructional Strategy	Conceptual Mean	Procedural Mean	Combined Mean
Level 1	CVHPI	3.73	3.86	3.76
	TVHPI	3.76	3.81	3.77
Level 2	CVHPI	3.76	3.81	3.80
	TVHPI	3.81	3.70	3.72
Level 3	CVHPI	2.19	2.17	2.17
	TVHPI	2.21	2.32	2.28
Level 4	CVHPI	1.60	1.66	1.63
	TVHPI	1.55	1.63	1.59

From Table 3, learners performed exceptionally well at Foundational levels 1 and 2 under both strategies, with the combined mean Scores exceeding 3.7. Both Strategies effectively supported conceptual and procedural understanding of simpler geometric tasks. Performance declined significantly as tasks became more complex at level 3.

TVHPI demonstrated a slight advantage in procedural understanding, with a combined mean of 2.28 compared to 2.17 for CVHPI. This suggests that Dynamic visualization tools in TVHPI may support intermediate-level reasoning. The lowest performance was observed at Level 4, where abstract reasoning is required. Combined means dropped to 1.63 (CVHPI) and

1.59 (TVHPI). The slight advantage of CVHPI might be its repetitive and structured approach, which could offer additional support for complex tasks.

Furthermore, to determine if the observed differences in performance across levels were statistically significant, a Kruskal-Wallis test was conducted on the combined scores, and the results yielded a Kruskal-Wallis (H) Statistic of 453.32 and a p-value of 0.00. The significant p-value (< 0.05) confirms that performance differences across the four Van Hiele levels are statistically significant. This finding validates the observation that learners encounter increasing challenges as they progress to higher levels of geometric reasoning.

The findings also confirm that learners perform well at foundational levels but face challenges as tasks become more abstract. The findings answer the third research question, highlighting that foundational reasoning (Levels 1 and 2) benefits equally from both strategies. At the same time, TVHPI supports procedural understanding at Level 3, and CVHPI marginally aids complex reasoning at Level 4. Figure 3 further illustrates a Mapping of Mean Scores across Van Hiele Levels and Understanding types.

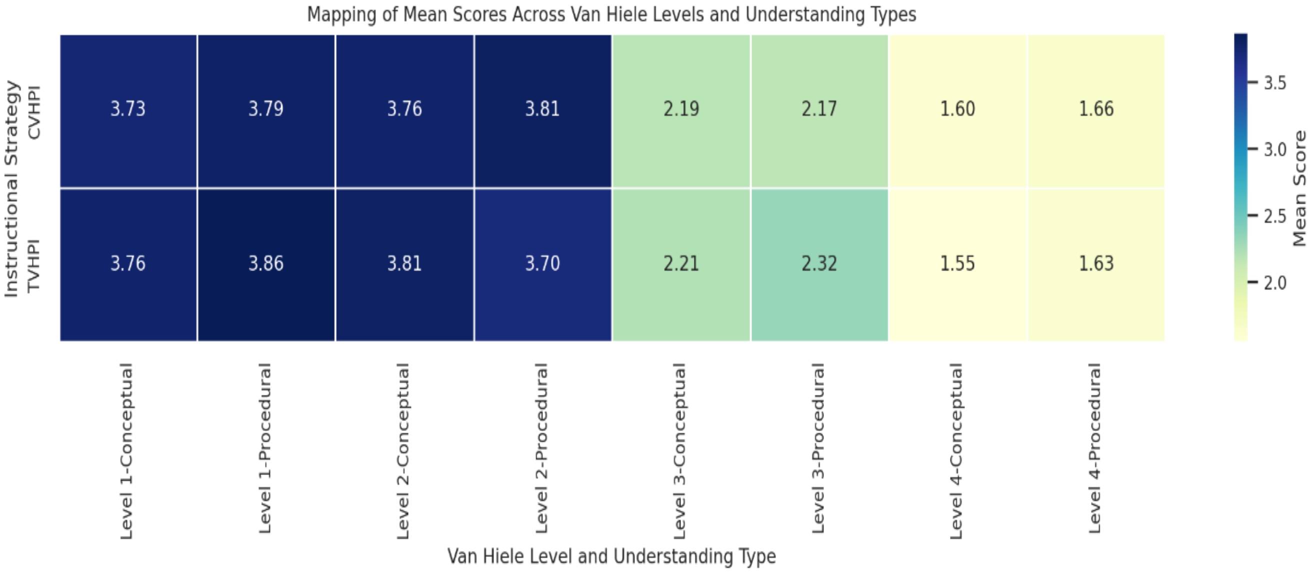

Figure 3: A Heatmap Mapping of Mean Scores across Van Hiele Levels and Understanding Types

Figure 3 depicts a Heatmap that maps the average performance of learners in each instructional strategy (TVHPI and CVHPI) across conceptual and procedural tasks at all four Van Hiele levels. Darker shades correspond to higher mean scores. The map reveals that learners under the TVHPI condition outperformed their CVHPI counterparts across almost all combinations of level and understanding type. This is especially evident in the earlier levels (Level 1 and 2), where learners showed strong foundational understanding. However, the relative drop in Level 4 conceptual tasks for both groups suggests that formal deductive reasoning remains challenging regardless of the strategy used. The Heatmap is a useful visual tool for identifying specific strengths and weaknesses in learner outcomes, providing actionable insights for instructional planning.

5. Discussions

The findings demonstrate the significant impact of technology-enhanced Van Hiele Phased Instruction on improving learners' conceptual understanding, procedural fluency, and performance across Van Hiele levels. This aligns with the growing research on technology's role in mathematics education, as discussed below.

5.1 How does TVHPI influence learners' conceptual understanding compared to CVHPI?

The results demonstrated that learners in the TVHPI group exhibited significantly higher conceptual understanding than those in the CVHPI group. This finding is strongly aligned with Faruk and Ozturk (2013), who reported that GeoGebra significantly improved students' comprehension of trigonometry and slope. Similarly, GeoGebra's dynamic and interactive environment encourages learners to visualize relationships and manipulate geometric figures, fostering a more profound understanding.

This aligns with the broader literature on conceptual understanding. For example, Kusmaryono and Suyitno (2016) observed that constructivist approaches with scientific methods enhanced conceptual understanding, particularly for learners with low initial competencies. Likewise, Kasmer and Kim (2011) demonstrated that prediction-based teaching strategies significantly improved conceptual understanding of algebraic functions. Faruk and Ozturk (2013) caution that technology alone may not lead to substantial gains across all Van Hiele levels despite these positive findings. Similarly, Eisenhart et al. (1993) observed that systemic constraints often prevent teachers from fully leveraging conceptual teaching, emphasizing the need for structured support.

The slightly lower variability in scores within the TVHPI group highlights the benefits of technology in reducing learning disparities. However, this echoes the limitations noted by Crooks and Alibali (2014), who argued that inconsistency in defining and measuring conceptual knowledge could complicate interpretations of such findings. Aligning tasks with theoretical definitions remains a critical challenge. The findings emphasize the need for structured teacher training programs, as Mudaly (2014) suggested, to ensure that teachers effectively integrate GeoGebra into their instructional practices. Additionally, future research should investigate long-term impacts, as recommended by Faruk and Ozturk (2013), to determine whether the observed benefits are sustained over time.

5.2 How does TVHPI affect learners' procedural fluency compared to CVHPI?

The current study found that TVHPI significantly enhanced procedural fluency compared to CVHPI. This is consistent with the work of Kanive et al. (2014), who reported that computer-based practice interventions significantly improved computational fluency and retention. Similarly, Asfar and Asfar (2020) found that integrating case-based learning with technological tools like Quizizz enhanced procedural understanding and engagement.

While these findings align with studies advocating technology-enhanced instruction, they also highlight its limitations. For instance, Ploger and Hecht (2009) noted that tools like Chartworld improved procedural fluency but required careful teacher guidance to ensure meaningful application. Likewise, Chappell and Killpatrick (2003) emphasized that procedural skill improvement should not come at the expense of conceptual depth, an important consideration for balanced instruction.

The lower variability in scores under TVHPI suggests that technology helps bridge performance gaps. This finding resonates with the work of Salim and Gilar (2020), who observed that balanced instruction incorporating both conceptual and procedural elements enhances overall problem-solving abilities. However, the results also align with Mutambara et al. (2020), who argued that preservice teachers often struggle with procedural fluency due to misconceptions, underscoring the need for targeted training.

The results underscore the importance of combining TVHPI with explicit instructional scaffolds, as Lestari and Surya (2017) recommended, to support learners with varying levels of procedural competency. Further research should examine the integration of procedural and conceptual knowledge, as Star (2000) suggested, to develop a coherent instructional framework.

5.3 At which Van Hiele levels do learners perform better in conceptual and procedural tasks under TVHPI and CVHPI?

Performance across Van Hiele levels revealed that TVHPI and CVHPI were effective at foundational levels (Levels 1 and 2). However, performance declined significantly at higher levels, with TVHPI demonstrating a slight advantage in procedural understanding at Level 3, while CVHPI marginally supported complex reasoning at Level 4. These findings align with Faruk and Ozturk (2013), who reported that GeoGebra improved comprehension but did not guarantee progression through advanced Van Hiele levels. This trend is consistent with Malatjie and Machaba (2019), who found that concept mapping improved learners' understanding of transformation geometry but required substantial teacher support. Similarly, Mendezabal and

Tindowen (2018) highlighted that while technology enhances skills and attitudes, traditional methods sometimes offer structure beneficial for abstract reasoning tasks.

The challenges at advanced levels reflect systemic issues Eisenhart et al. (1993) identified, including limited teacher expertise and curriculum constraints. Additionally, Richland et al. (2012) noted that procedural focus often undermines relational thinking, a critical component of higher-level geometric reasoning. The findings suggest a need for differentiated instructional strategies tailored to specific Van Hiele levels. For foundational levels, technology-enhanced tools like GeoGebra should be prioritized, as recommended by Irfan Taufan Asfar et al. (2018). For advanced levels, a hybrid approach combining the structured supports of CVHPI with the dynamic visualization capabilities of TVHPI may be more effective.

5.4 Broader Implications and Future Directions

The findings of this study align with the broader literature emphasizing the transformative potential of technology in mathematics education. As Al-Mutawah et al. (2019) suggested, integrating conceptual tasks early in education and providing multiple representations can strengthen understanding. However, as Faruk Tutkun and Ozturk (2013) noted, successful implementation requires addressing challenges such as teacher preparedness and resource access. Future research should explore long-term impacts, as recommended by Faruk Tutkun and Ozturk (2013) and Mendezabal and Tindowen (2018), to understand how consistent use of TVHPI influences learning outcomes across diverse contexts. Additionally, the role of teacher training, as highlighted by Mudaly (2014) and Chappell and Killpatrick (2003), remains critical for optimizing the benefits of technology-enhanced instruction.

6. Conclusion

This study examined the impact of TVHPI on learners' conceptual understanding, procedural fluency, and performance across Van Hiele levels of geometric reasoning compared to CVHPI. The findings revealed that TVHPI significantly enhances conceptual and procedural learning outcomes, offering a dynamic, interactive environment for engaging with geometric concepts. Both instructional strategies proved effective at foundational Van Hiele levels; however, TVHPI demonstrated a slight procedural advantage at intermediate levels, while CVHPI marginally supported complex reasoning at advanced levels. therefore, we can assume the study contributes to the growing evidence supporting integrating dynamic tools like GeoGebra in mathematics education. TVHPI offers a promising avenue for fostering conceptual understanding and procedural fluency by effectively addressing foundational and intermediate reasoning. However, achieving sustained progress across all Van Hiele levels will require a balanced approach that integrates technology with structured pedagogical practices.

7. Recommendations

This study recommends integrating technology-enhanced instructional strategies, such as GeoGebra, into mathematics education to improve conceptual understanding and procedural fluency. Especially integrating GeoGebra into the CVHPI approaches can maximize learning outcomes, mainly for complex geometric reasoning tasks. Teachers should receive targeted professional development to implement these strategies effectively, and curricula should be designed to align instructional tasks with learners' Van Hiele levels.

8. Limitations of the study

While contextually focused, this study has limitations that provide opportunities for future research to explore the impact of TVHPI in diverse settings. The short intervention duration highlights the potential for long-term studies to assess sustained effects on learning. While balancing baseline scores, the purposive sampling approach invites further exploration using randomized designs. Additionally, resource availability and teacher readiness challenges present opportunities to innovate scalable and accessible solutions for broader implementation.

Data Availability: The data supporting this study's findings can be accessed on request.

Conflict of Interest: On behalf of all authors, the corresponding author states that there is no conflict of interest.

Funding Statement: The study did not receive any funding

Doctoral Research Attribution: This article is part of the first author's doctoral study that focuses on the interplay among instructional strategies, learners' geometry attitudes, and Van Hiele's geometric reasoning levels in selected Midwestern Uganda secondary schools.

Ethical Considerations:

Ethical approval was obtained from the relevant institutional review boards to ensure compliance with research standards. Following this, permission was obtained from the district education departments and the headteachers of the participating schools. Teachers involved in implementing the intervention provided their consent to participate in training, teaching, assessment, and observation activities. During data collection, which included pre-tests, post-tests, and lesson observations, learners were fully briefed on the study's objectives and assured that their participation would not result in any physical or mental harm. Informed consent was obtained from the participating children's parents or teachers (acting as legal guardians). Informed consent was obtained directly from participants aged 18 and above. To ensure technology fairness, learners exposed to the CVHPI were allowed to interact with the GeoGebra from the computers for one week.

References

Al-Mutawah, M. A., Thomas, R., Eid, A., Mahmoud, E. Y., & Fateel, M. J. (2019). Conceptual understanding, procedural knowledge, and problem-solving skills in mathematics: High school graduates' work analysis and standpoints. *International Journal of Educationtiontion and Practice*, *7*(3), 258–273. https://doi.org/10.18488/journal.61.2019.73.258.273

Alkhateeb, M. A., & Al-Duwairi, A. M. (2019). The Effect of Using Mobile Applications (GeoGebra and Sketchpad) on the Students' Achievement. *International Electronic Journal of Mathematics Education*, *14*(3), 523–533. https://doi.org/10.29333/iejme/5754

Asfar, A. M. I. T., & Asfar, A. M. I. A. (2020). Case-based games learning strategies to improve conceptual understanding in mathematics. *Journal of Physics: Conference Series*, *1663*(1). https://doi.org/10.1088/1742-6596/1663/1/012060

Bartell, T. G., Webel, C., Bowen, B., & Dyson, N. (2013). Prospective teacher learning: Recognizing evidence of conceptual understanding. *Journal of Mathematics Teacher Education*, *16*(1), 57–79. https://doi.org/10.1007/s10857-012-9205-4

Bekene Bedada, T., & Machaba, F. (2022). The effect of GeoGebra on STEM students' learning trigonometric functions. *Cogent Education*, *9*(1). https://doi.org/10.1080/2331186X.2022.2034240

Bekene, T. (2020). *Implementation of GeoGebra, a Dynamic Mathematical Software for Teaching and Learning of Calculus in Ethiopia. Author: Tola Bekene, Wachemo University, Ethiopia*. *11*(9), 838–860.

Bossé, M. J., & Bahr, D. L. (2008). The State of Balance Between Procedural Knowledge and Conceptual Understanding in Mathematics Teacher Education. *International Journal for Mathematics Teaching and Learning*, 11–25. https://scholarsarchive.byu.edu/facpubhttp://www.cimt.plymouth.ac.uk/journal/default.htm.

Chappell, K. K., & Killpatrick, K. (2003). Effects of concept-based instruction on students' Conceptual understanding and procedural knowledge of calculus. *Problem, Resources, and Issues in Mathematics Undergraduate Studies*, *13*(1), 17–37. https://doi.org/10.1080/10511970308984043

Crooks, N. M., & Alibali, M. W. (2014). Defining and measuring conceptual knowledge in mathematics. *Developmental Review*, *34*(4), 344–377. https://doi.org/10.1016/j.dr.2014.10.001

Crowley, M. (1987). The van Hiele model of the development of geometric thought. *Learning and Teaching Geometry, K-12*, 1–16. http://www.csmate.colostate.edu/docs/math/mathactivities/june2007

Edulsa, V. A. (2022). Conceptual Understanding and Academic Performance of Students in Mathematics. *Psych Educ*, *5*, 736–745. https://doi.org/10.5281/zenodo.7352844

Eisenhart, M., Borko, H., Underhill, R., Brown, C., Jones, D., & Agard, P. (1993). Conceptual Knowledge Falls through the Cracks: Complexities of Learning to Teach Mathematics for Understanding. *Journal for Research in Mathematics Education*, *24*(1), 8. https://doi.org/10.2307/749384

Faruk, T. O., & Ozturk, B. (2013). The effect of GeoGebra mathematical software on academic success and Van Hiele geometrical thinking level. *International Journal of Academic Research*, *5*(4), 22–28. https://doi.org/10.7813/2075-4124.2013/5-4/b.3

Geiger, V., Forgasz, H., Tan, H., Calder, N., & Hill, J. (2012). Technology in mathematics education. *Research in Mathematics Education in Australasia 2008-2011*, *9789460919*, 111–141. https://doi.org/10.1007/978-94-6091-970-1_7

Irfan Taufan Asfar, A. M., Iqbal Akbar Asfar, A. M., Darmawati, & Darmawan, D. (2018). The Effect of REACE (Relating, Exploring, Applying, Cooperating, and Evaluating) Learning Model Toward the Understanding of Mathematics Concept. *Journal of Physics: Conference Series*, *1028*(1). https://doi.org/10.1088/1742-6596/1028/1/012145

Kanive, R., Nelson, P. M., Burns, M. K., & Ysseldyke, J. (2014). Comparison of the effects of computer-based practice and conceptual understanding interventions on mathematics fact retention and generalization. *Journal of Educational Research*, *107*(2), 83–89. https://doi.org/10.1080/00220671.2012.759405

Kasmer, L., & Kim, O. K. (2011). Using Prediction to Promote Mathematical Understanding and Reasoning. *School Science and Mathematics*, *111*(1), 20–33. https://doi.org/10.1111/j.1949-8594.2010.00056.x

Khalil, M., Khalil, U., & ul Haq, Z. (2019). Geogebra as a Scaffolding Tool for Exploring Analytic Geometry Structure and Developing Mathematical Thinking of Diverse Achievers. *International Electronic Journal of Mathematics Education*, *14*(2), 427–434. https://doi.org/10.29333/iejme/5746

Kusmaryono, I., & Suyitno, H. (2016). The Effect of Constructivist Learning Using a Scientific Approach on Mathematical Power and Conceptual Understanding of Students in Grade IV. *Journal of Physics: Conference Series*, *693*(1). https://doi.org/10.1088/1742-6596/693/1/012019

Kusumaningsih, W., Saputra, H. A., & Aini, A. N. (2019). Cognitive style and gender differences in a conceptual understanding of mathematics students. *Journal of Physics: Conference Series*, *1280*(4). https://doi.org/10.1088/1742-6596/1280/4/042017

Lestari, L., & Surya, E. (2017). The effectiveness of a realistic mathematics education approach on students' mathematical concept understanding ability. *International Journal of Sciences: Basic and Applied Research*, *34*(1), 91–100. http://gssrr.org/index.php?journal=JournalOfBasicAndApplied

Machisi, E., & Feza, N. N. (2021). Van Hiele Theory-Based Instruction and Grade 11 Students' Geometric Proof Competencies. *Contemporary Mathematics and Science Education*, *2*(1), ep21007. https://doi.org/10.30935/conmaths/9682

Malatjie, F., & Machaba, F. (2019). Exploring mathematics learners' conceptual understanding of coordinates and transformation geometry through concept mapping. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(12), 1–16. https://doi.org/10.29333/EJMSTE/110784

Mendezabal, M. J. N., & Tindowen, D. J. C. (2018). Improving students' attitude, conceptual understanding, and procedural skills in differential Calculus through Microsoft Mathematics. *Journal of Technology and Science Education*, *8*(4), 385–397. https://doi.org/10.3926/jotse.356

Mollakuqe, V., Rexhepi, S., & Iseni, E. (2020). Incorporating Geogebra into Teaching Circle Properties at High School Level and Its Comparison with the Classical Method of Teaching. *International Electronic Journal of Mathematics Education*, *16*(1), em0616. https://doi.org/10.29333/iejme/9283

Moru, E. K., Malebanye, M., Morobe, N., & George, M. J. (2021). A Van Hiele Theory analysis for teaching the volume of three-dimensional geometric shapes. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, *6*(1), 17–31. https://doi.org/10.23917/jramathedu.v6i1.11744

Mosese, N., & Ogbonnaya, U. I. (2021). GeoGebra and students' learning achievement in trigonometric functions, graphs, representations, and interpretations. *Cypriot Journal of Educational Sciences*, *16*(2), 827–846. https://doi.org/10.18844/CJES.V16I2.5685

Mudaly, V. (2014). A visualisation-based semiotic analysis of learners' conceptual understanding of graphical functional relationships. *African Journal of Research in Mathematics, Science and Technology Education*, *18*(1), 3–13. https://doi.org/10.1080/10288457.2014.889789

Mukamba, E., & Makamure, C. (2020). Integration of GeoGebra in Teaching and Learning Geometric Transformations at Ordinary Level in Zimbabwe. *Contemporary Mathematics and Science Education*, *1*(1), ep20001. https://doi.org/10.30935/conmaths/8431

Mutambara, L. H. N., Tendere, J., & Chagwiza, C. J. (2020). Exploring the conceptual understanding of the quadratic function concept in teachers' colleges in Zimbabwe. *Eurasia Journal of Mathematics, Science and Technology Education*, *16*(2), 1–17. https://doi.org/10.29333/ejmste/112617

Narh-kert, M., & Sabtiwu, R. (2022). Use of GeoGebra to Improve Performance in Geometry. *African Journal of Educational Studies in Mathematics and Sciences*, *18*(1), 29–36.

Ndungo, I. (2024). An Assessment of the Alignment of Transformation Geometry with Van Hiele Levels in Uganda's Lower Secondary Mathematics Curriculum. *Turkish Journal of Mathematics Education*, *5*(2), 33–41. https://tujme.org/index.php/tujme/article/view/108

Ndungo, I., Akugizibwe, E., & Balimuttajjo, S. (2024). Analyzing trends and suggested instructional strategies for Geometry education : Insights from Uganda Certificate of Education-Mathematics Examinations, 2016-2022. *African Journal of Teacher Education*, *13*(2), 153–186. https://journal.lib.uoguelph.ca/index.php/ajote/article/view/8106

Ndungo, I., Nazziwa, C., Kabiswa, W., Opio, P., Akugizibwe, E., & Mpungu, K. (2025). Enhancing STEM Instruction through Indigenous Materials and ICT Integration : A Critical Assessment of Teachers ' Knowledge, Experiences, Attitudes, and Readiness. *East African Journal of Education Studies*, *8*(2), 132–149. https://doi.org/10.37284/eajes.8.2.2872

Ngwabe, A., & Felix, C. (2020). Using GeoGebra to Address Students' Misconceptions about the Transformation of Algebraic Hyperbola Functions. *African Journal of Research in Mathematics, Science and Technology Education*, *24*(3), 348–360. https://doi.org/10.1080/18117295.2020.1854494

Ploger, D., & Hecht, S. (2009). Enhancing children's conceptual understanding of mathematics through ChartWorld software. *Journal of Research in Childhood Education*, *23*(3), 267–277. https://doi.org/10.1080/02568540909594660

Praveen, S., & Leong, K. E. (2013). Effectiveness of Using GeoGebra on Students ' Understanding in Learning Circles. *The Malaysian Online Journal of Educational Technology*, *1*(4), 1–11.

Pujawan, I. G. N., Suryawan, I. P. P., & Prabawati, D. A. A. (2020). The effect of van Hiele's learning model on students' spatial abilities. *International Journal of Instruction*, *13*(3), 461–474. https://doi.org/10.29333/iji.2020.13332a

Richland, L. E., Stigler, J. W., & Holyoak, K. J. (2012). Teaching the Conceptual Structure of Mathematics. *Educational Psychologist*, *47*(3), 189–203. https://doi.org/10.1080/00461520.2012.667065

Salim, N. D., & Gilar, J. M. (2020). Conceptual Understanding and Procedural Knowledge: A Case Study on Learning Mathematics of Fractional Material in Elementary School. *Journal of Physics: Conference Series*, *1477*(4). https://doi.org/10.1088/1742-6596/1477/4/042037

Star, J. R. (2000). On the Relationship Between Knowing and Doing in Procedural Learning. *Proceedings of the Twenty-First Annual Conference of the Cognitive Science Society*, 818–818. https://doi.org/10.4324/9781410603494-175

Sunzuma, G. (2023). Technology integration in geometry teaching and learning: A systematic review (2010-2022). *Lumat*, *11*(3). https://doi.org/10.31129/LUMAT.11.3.1938

Usiskin, Z. (1982). *Van Hiele Levels and Achievement in Secondary School Geometry: Final report of the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project* (p. 227).

Zengin, Y. (2017). The effects of GeoGebra software on pre-service mathematics teachers' attitudes and views toward proof and proving. *International Journal of Mathematical Education in Science and Technology*, *48*(7), 1002–1022. https://doi.org/10.1080/0020739X.2017.1298855