

TESTING FOR LINEAR ENGEL CURVES An Application of NLES to Cross-country Data

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The demand system, NLES, proposed recently by Blundell and Ray (1982), is shown to aggregate consistently across households. It is then used to analyse time series expenditure data of Korea, Greece, Israel and Puerto Rico. The empirical results reject linear Engel curves for each country.

1. Introduction

In a recent paper in this journal, Blundell and Ray (1982) proposed and estimated a new generalisation of the LES, namely, NLES, that allows for both non-linear Engel curves and non-separable preferences. The proposed system, besides being a flexible functional form, has the attraction of providing for a framework which allows for separate tests for non-linear Engel curves *and* non-separable preferences in a manner not possible in other comparable LES generalisations, for example the QES proposed by Pollak and Wales (1978). Detailed empirical evidence on NLES as applied to U.K. budget and time series data is presented in Blundell and Ray (1983).

In the present paper, we show that NLES is also a member of the PIGL family as defined in Deaton and Muellbauer (1980) and, hence, satisfies the requirement of consistent aggregation implied by the concept of the 'representative' consumer. In this regard, therefore, NLES has the same attractive feature as the 'almost ideal demand system' (AIDS)

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proposed by Deaton and Muellbauer. However, while AIDS is a member of PIGLOG which is a limiting case of PIGL, NLES is a member of PIGL itself. AIDS and NLES are non-nested to one another and while NLES can nest LES as a special case, AIDS does not. In the present paper, we also provide further evidence on the usefulness of NLES by using it to analyse the expenditure patterns of Korea, Greece, Israel and Puerto Rico.

The plan of the paper is as follows. Section 2 demonstrates that NLES is a member of PIGL. Section 3 describes the data and presents the results of estimation. We end with the concluding note of section 4.

2. NLES: A member of PIGL

The NLES cost function is given by

$$C(p, u) = [a(p, \alpha) + b(p, \alpha)u]^{1/\alpha}, \quad (1)$$

where a, b are homogeneous of degree α in price p . NLES is obtained on choice of following functional forms for a, b :

$$a(p, \alpha) = \sum_i \sum_j \gamma_{ij}^* p_i^{\alpha/2} p_j^{\alpha/2}, \quad \alpha > 0, \quad (2a)$$

$$b(p, \alpha) = \prod_k p_k^{\beta_k \alpha}. \quad (2b)$$

The application of Shephard's Lemma to (1), (2a) and (2b) gives NLES which can be written in budget share form for household h as

$$W_{ih} = \sum_j \gamma_{ij} \left(\frac{p_i}{\mu_h} \right)^{\alpha/2} \left(\frac{p_j}{\mu_h} \right)^{\alpha/2} + \beta_i \left\{ 1 - \sum_i \sum_j \gamma_{ij} \left(\frac{p_i}{\mu_h} \right)^{\alpha/2} \left(\frac{p_j}{\mu_h} \right)^{\alpha/2} \right\},$$

$$h = 1, \dots, H, \quad (3)$$

where $\gamma_{ij} [= (\gamma_{ij}^* + \gamma_{ji}^*)/2]$ is symmetric, μ_h = total expenditure, $\sum \beta_i = 1$, and H denotes the total number of households. The parameter α measures the non-linearity in Engel curves generated by the data and is unity when preferences are linear.

If W_i denotes the 'average' budget share of item i for all households

and assuming that all households face identical prices at a given time, then

$$\begin{aligned} \bar{W}_i &= \sum_{h=1}^H \frac{\mu_h}{\sum \mu_h} W_{ih} \\ &= \sum_{h=1}^H \frac{\mu_h}{\sum \mu_h} \left[\sum_j \gamma_{ij} \left(\frac{p_i^{\alpha/2} p_j^{\alpha/2}}{\mu_h^\alpha} \right) + \beta_i \left\{ 1 - \sum_i \sum_j \gamma_{ij} \left(\frac{p_i^{\alpha/2} p_j^{\alpha/2}}{\mu_h^\alpha} \right) \right\} \right] \quad (4) \end{aligned}$$

$$= \sum_j \gamma_{ij} p_i^{\alpha/2} p_j^{\alpha/2} \sum_h \frac{\mu_h^{1-\alpha}}{\sum \mu_h} + \beta_i \left\{ 1 - \sum_i \sum_j \gamma_{ij} p_i^{\alpha/2} p_j^{\alpha/2} \sum_h \frac{\mu_h^{1-\alpha}}{\sum \mu_h} \right\}, \quad \text{i.e.,}$$

$$\bar{W}_i = \sum_j \gamma_{ij} p_i^{\alpha/2} p_j^{\alpha/2} \cdot \frac{1}{\mu_0^\alpha} + \beta_i \left\{ 1 - \sum_i \sum_j \gamma_{ij} p_i^{\alpha/2} p_j^{\alpha/2} \cdot \frac{1}{\mu_0^\alpha} \right\}, \quad \text{where} \quad (5)$$

$$\mu_0 = \left[\sum_h \left(\mu_h^{1-\alpha} / \sum_h \mu_h \right) \right]^{-1/\alpha} \quad (5')$$

(5) can be re-written as

$$\bar{W}_i(\mu_1, \dots, \mu_H, p) = \beta_i + \eta_i(p, \alpha) V(\mu_0), \quad \text{where} \quad (6)$$

$$\sum \beta_i = 1, \quad \eta_i(p, \alpha) = \sum_j \gamma_{ij} p_i^{\alpha/2} p_j^{\alpha/2} - \beta_i \sum_i \sum_j \gamma_{ij} p_i^{\alpha/2} p_j^{\alpha/2},$$

$$\sum_i \eta_i(p, \alpha) = 0.$$

$V(\mu_0) = \mu_0^{-\alpha}$ where μ_0 depends on the expenditure distribution of the individual households and is given by (5'). It is worth noting that if $\alpha = 1$, i.e., if Engel curves are linear, μ_0 will only depend on $\bar{\mu}$, namely the mean of the individual expenditures. (6), being of the form taken by PIGL demand systems [see Deaton and Muellbauer (1980, ch. 6)], we have shown that NLES is a member of PIGL and, hence, aggregates consistently across households.

3. Data, estimation and results

The data base is annual time series expenditure data of Korea, Greece, Israel and Puerto Rico obtained from *UN Statistical Yearbook* [United Nations (1960–1979)]. The aggregate expenditures in the individual countries were all converted into *per capita* figures in U.S. dollars using exchange rates in the base year 1970. Each of these countries has 25 observations covering the identical time period 1953–1977. A 4-commodity breakdown was used: (a) Food (includes alcohol and beverages), (b) Clothing, (c) Gross Rent, and (d) Miscellaneous Goods (includes transport, education and health, entertainment, etc.).

The present paper reports the results of estimating eq. (3) with $\gamma_{ij} = 0$, $i \neq j$ enforced. The resulting system still allows non-separable preferences via α , though it is more likely to capture non-linearity in Engel curves, which is the central interest in this paper.¹ The estimation method was Full Information Maximum Likelihood using the RESIMUL program of Wymer (1973). α was estimated by grid search.

The log-likelihood values of NLES and LES for the four countries are presented in table 1. Since in this paper, NLES specialises to LES when $\alpha = 1$, a likelihood based Chi-square test (one degree of freedom) suggests rejection of linear Engel curves for each of the four countries. This is confirmed by the point estimate of α . It is also interesting to observe that the richer a country as measured by its *per capita* income, the more non-linear are its preference patterns. This latter result may well be explained by the fact that, on the present data set at least, the variance of expenditure in Puerto Rico is far higher than that in Korea.

Table 1
Log-likelihood values.

Country	L-1: NLES; $k = 8$	L-1: LES; $k = 7$ ^a	$\hat{\alpha}$	<i>Per capita</i> income ^b
Korea	312.42	292.747	0.57	266
Greece	402.273	397.183	0.52	1134
Israel	340.162	331.494	0.43	1886
Puerto Rico	345.223	338.406	0.32	2089

^a k is the number of parameters.

^b US\$ (1970).

¹ Empirical results on the more general system (3) and on more cross-country data will be presented in Kasekende (1983) currently in preparation.

Table 2
Expenditure levels and MBS for Food.^a

Country		1956	1963	1970	1977
Korea	<i>EXP</i>	\$122.8	\$111.769	\$184.81	\$597.21
	<i>MBS</i>	0.5042	0.496	0.466	0.454
	LES		0.460	(0.0092351)	
Greece	<i>EXP</i>	\$248	\$391.9	\$745.3	\$1823.66
	<i>MBS</i>	0.346	0.329	0.306	0.303
	LES		0.323	(0.0081299)	
Israel	<i>EXP</i>	\$523.06	\$666.06	\$1031.26	\$1469.9
	<i>MBS</i>	0.321	0.289	0.288	0.253
	LES		0.2778	(0.008205)	
Puerto Rico	<i>EXP</i>	\$420.49	\$771.34	\$1437.68	\$2554.14
	<i>MBS</i>	0.286	0.217	0.204	0.208
	LES		0.2326	(0.0179428)	

^a *EXP* is the expenditure level *per capita*, *MBS* is the marginal budget share estimated by NLES ($\alpha \neq 1$, $\gamma_{ij} = 0$, $i \neq j$). The figures in parentheses are standard errors of estimate.

The marginal budget share (MBS) for Food implied by the NLES parameter estimates for each country are presented in table 2 at four different years. The corresponding *per capita* total expenditure in US\$ are also presented, along with the MBS for Food implied by LES (standard errors in parentheses). The results generally indicate an inverse relationship between the level of expenditure and MBS for Food both across countries and over time in conformity with Engel's Law. The results also show that in each country, the LES over-estimates the MBS for Food when *per capita* expenditure is at relatively high levels (as in 1977) and significantly under-estimates it at lower levels (as in 1956).

4. Conclusion

The present paper provides evidence in favour of the demand system NLES proposed recently in Blundell and Ray (1982,1983). We demonstrate that NLES is a member of the PIGL family and hence aggregates 'consistently' across households. This is a particularly attractive property for a demand system to possess, if estimating on grouped budget data which are available rather than individual, micro data. The empirical

results demonstrate the usefulness of NLES in analysing cross-country expenditure data to test for non-linear Engel curves on time series data.

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