

Research

Application of interactive software in classrooms: a case of GeoGebra in learning geometry in secondary schools in Uganda

Marjorie Sarah Kabuye Batiibwe¹ 

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Abstract

The incorporation of subject-specific technologies in teaching and learning mathematics has been extensively investigated in different areas outside Uganda. The benefits are extensive, although the adoption of these technologies has been threatened by teachers needing help to apply them effectively for instruction. While this alone would aptly justify a comprehensive assessment of its applicability on the local scene, the post-COVID-19 era dynamics escalated its urgency and relevance. Thus, third-year pre-service mathematics teachers' knowledge and aptitude to use GeoGebra in teaching and learning geometry were developed in this study. In addition, the consequence of using this software on students' achievement in geometry was investigated using a quantitative research approach. Ninety-four third-year pre-service mathematics teachers completed a three-week intensive capacity-building workshop using GeoGebra. Their self-ratings on their technological pedagogical content knowledge (TPACK) before and after the training were measured using a self-administered questionnaire. A total of 406 students from four schools in Central Uganda participated in the quasi-experiment, with 199 and 207 students assigned to the experimental and control groups, respectively. Pre- and post-achievement tests were meted out to the students. While descriptive statistics showed a positive change in the teachers' technological knowledge (TK), technological pedagogical knowledge (TPK), technological content knowledge (TCK), and TPACK, independent samples *t*-tests revealed a substantial difference in the post-test mean scores between the two groups since the experimental group students outshone those in the control group. These findings are consistent with global literature, and thus, schools in Uganda should adopt the use of GeoGebra through a continuous capacity-building approach, while considering the challenges and limitations associated with the software.

Keywords GeoGebra · Geometry · Interactive software · Mathematics achievement · Pre-service mathematics teachers · TPACK

1 Introduction

Technology and mathematics instruction have long been complimentary, mainly using tools to ease arithmetic computations and the development of models. Tools like calculators and computers have been essential in developing this subject [47]. More recently, interactive web-based software has been designed as a teaching tool and is growing exponentially worldwide [45]. GeoGebra is one such tool. Marcus Hohenwater created this multilingual dynamic mathematics software in 2002 as a teaching platform focusing on Algebra, Geometry, Calculus, and Statistics [21]. It supports learners' engagement at all levels of mathematics, primarily highlighting relationships in concepts like Algebra and Geometry [16].

✉ Marjorie Sarah Kabuye Batiibwe, batiibwemarjorie@yahoo.co.uk | ¹Department of Science, Technical and Vocational Education School of Education College of Education and External Studies, Makerere University, P. O. Box 7062, Kampala, Uganda.



GeoGebra employs a comprehensible functional interface with tools that empower instructors to organize and present targeted learning goals for particular classes. For example, an instructor can use the GeoGebra calculator suite to plot 2-dimensional and 3-dimensional functions, equations, and curves. The platform's interactive abilities allow for sharing, testing, marking, and giving feedback individually in class. Importantly, GeoGebra is an open source, freely available, and secure through multi-factor authentication [11].

The advantages that accrue to incorporating technology into teaching and learning have been shown by several scholars to have overarching benefits. For example, Wassie and Zergaw [51] asserted that incorporating technology during teaching and learning mathematics addresses students' learning necessities and interests. Ginting [18] argued that it influences students' learning styles since they can see, touch, and familiarize themselves with the topics they confront in school. Meanwhile, Abramovich [1] asserted that through modeling, simulation, and visualization, technology makes it feasible for teachers and students to communicate typically abstract mathematical ideas in conceptually affluent and comprehensible ways. Thus, technology allows students to build their knowledge and discover new strategies for problem-solving [13]. However, Spangenberg and De Freitas [44] weighed these benefits and indicated that numerous mathematics teachers successfully grapple with incorporating technology into their teaching and therefore require continuous professional development programs to improve their technological pedagogical content knowledge (TPACK).

It is in consideration of such benefits and threats that the National Council of Teachers of Mathematics (NCTM) [23] in the United States, together with numerous scholars such as Pepin, Xu, Trouche, and Wang [40], Arthur et al. [5], and König, Jäger-Biela, and Glutsch [28], maintained that teachers require a continuous profound awareness of the mathematical technologies at their disposal to entirely show their potentials. Contextually, in the wake of COVID-19, pre-service mathematics teachers in Uganda, as part of their training, were expected to complete their school practice through a blended approach, using both online and face-to-face (F2F) means. This had to involve teaching secondary school mathematics to students at this level through both online means, an approach they had never encountered before, and the usual physical means when necessary and conducive. Radović, Marić, and Passey [41] asserted that mathematics teachers need support and guidance before adopting any technological tool. Thus, the pre-service mathematics teachers needed to be prepared for this new experience by exposing them to mathematical technologies at their disposal, thereby enhancing their technological knowledge (TK) and understanding. Such understanding involves mathematics teachers' ability to design tasks, for example, those that are shown by Jupri and Sispiyati [24], that utterly engross students in learning mathematics. Besides, they need more than TK, as highlighted by Davies and West [15], that technology-aided teaching entails confidence or belief in oneself, impetus, and astute incorporation. Consequently, this study aimed to enhance third-year pre-service mathematics teachers' knowledge and skills in using GeoGebra as a pedagogical tool to improve students' understanding of geometry. Although GeoGebra, as seen earlier, can be used to teach various components of mathematics [21], the choice of geometry was premised on the fact that according to the Uganda National Examinations Board (UNEB), most students fail geometry questions in national examinations, especially at the ordinary secondary school level [48]. Students find it difficult to visualize how 3- and 2-dimensional shapes, angles, and lines relate from different perspectives [25] and lack appropriate resources for learning geometry [19]. Thus, as the third-year pre-service mathematics teachers had not previously used GeoGebra for teaching and learning geometry, this study addressed the gap. In particular, the study was guided by the following research questions:

- (i) How does exposure to an intensive GeoGebra workshop impact the TPACK of pre-service mathematics teachers?
- (ii) What is the effect on learners' achievement when taught geometry using GeoGebra?

2 Theoretical framework

For teachers to successfully learn how to integrate technology in teaching secondary school geometry, they need a specialized form of knowledge conceptualized as Technological Pedagogical Content Knowledge (TPACK) [32]. Teachers can successfully teach and engage students with technology by using the insights that TPACK offers them about the interrelationships between their knowledge constructs. According to Mishra and Koehler [32], TPACK is one of the constructs of the TPACK framework. The TPACK framework describes seven knowledge constructs, namely Content Knowledge (CK), Pedagogical Knowledge (PK), Technological Knowledge (TK), Pedagogical Content Knowledge (PCK), Technological Pedagogical Knowledge (TPK), Technological Content Knowledge (TCK), and Technological Pedagogical Content Knowledge (TPACK) as shown in Fig. 1.

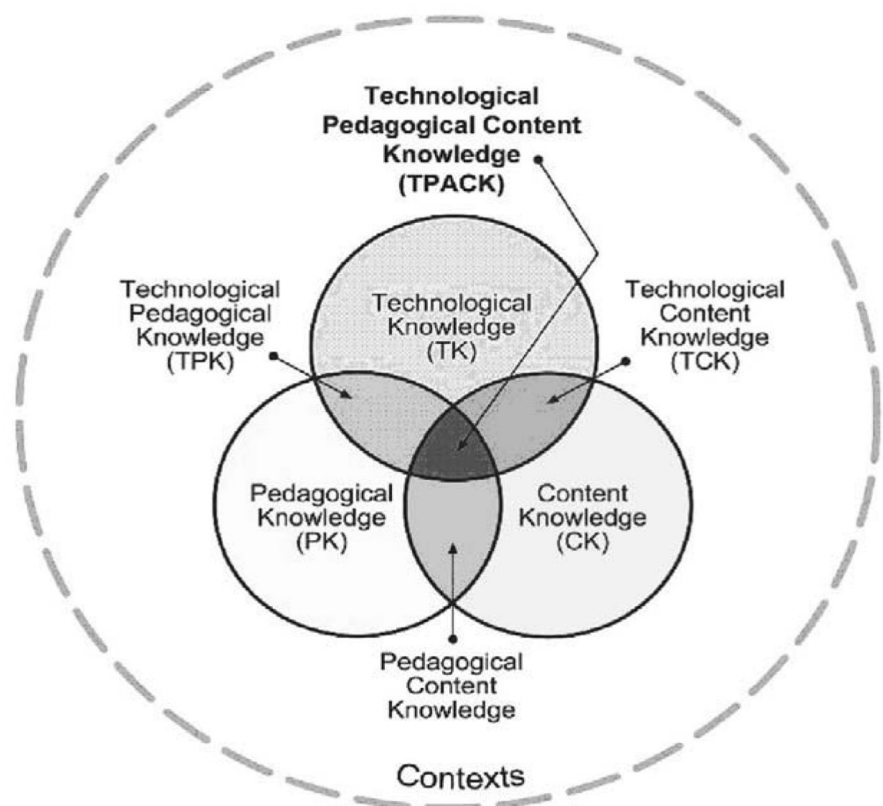
While Mishra and Koehler [32] defined CK as “the knowledge about the actual subject matter that is to be taught” (p.1026), they defined PK as the knowledge about the processes and practices or methods of teaching and learning and how it encompasses...overall educational purposes, values, and aims” (p.1026). In addition, they defined TK as the “knowledge about standard technologies, such as books, chalk, and blackboard, and more advanced technologies, such as the Internet and digital video” (p.1027); PCK as the “knowledge of pedagogy that applies to the teaching of specific content” (p.1027); TPK as the “knowledge of existence, components, and capabilities of various technologies as they are used in teaching and learning settings, and conversely, knowing how teaching might change as the result of using particular technologies” (p.1028); TCK as the “knowledge about how technology and content are reciprocally related” (p.1028); and TPACK as an “emergent form of knowledge that goes beyond all the three components; content, pedagogy, and technology” (p.1028). This framework, therefore, alludes that teachers ought to know about the relationships between technology, pedagogy, and content. In the context of this study, that is, the relationships between GeoGebra, how to teach geometry, and the geometry content. Thus, the TPACK framework guided this study.

3 Literature review

3.1 Teacher support in using GeoGebra through capacity-building

Capacity building is the process of imparting knowledge and developing skills while optimizing the advantages of participation and knowledge sharing. In the context of this study, capacity building was necessary to assist pre-service mathematics teachers in developing the specialized knowledge, TPACK, required to incorporate technology, such as GeoGebra, into the teaching and learning process. GeoGebra is among the most preferred effective mathematics software in many countries, such as South Africa [33] and Ghana [46]. Unlike other mathematics software, GeoGebra is web-based, freely available, and downloadable [51]. However, Uwurukundo, Maniraho, and Tusiime [50] asserted that even though GeoGebra appears to be predominantly efficient, such efficiency relies on how it is incorporated into the teaching and learning process. Thus, they argued that teacher knowledge is a pivotal factor in integrating GeoGebra. Mishra and Koehler [32] contend that teachers need an emergent form of knowledge that

Fig. 1 Technological pedagogical content knowledge (TPACK) as a framework for the use of technology in the teaching and learning process. Source: [32]



goes beyond content, pedagogy, and technology to successfully integrate technology into their teaching. This specialized knowledge they termed technological pedagogical content knowledge (TPACK). According to Mwingirwa and Miheso-O'Connor [35], if mathematics teachers are to acquire TPACK, they need systemic support to take advantage of technology and further sustain their engagement with it, at least at the classroom level.

Although GeoGebra has been shown to be effective and efficient in teaching mathematics, not all mathematics teachers at all education levels successfully utilize it in their teaching practices. For example, Garba [17] scrutinized issues and challenges that secondary school teachers in the Makurdi metropolis in Nigeria encountered while using GeoGebra in teaching and learning mathematics and showed that, among others, low teacher capability was among the challenges that averted teachers from regularly using GeoGebra in the process of teaching and learning mathematics. Thus, this finding supports Mwingirwa and Miheso-O'Connor's [35] assertion that mathematics teachers need support before they can incorporate technology such as GeoGebra in their classrooms. To this effect, scholars such as Kirikçilar and Yildiz [27], Açıkgül and Aslaner [2], Bhagat, Chang, and Huang [10], and Kartal and Çınar [26] investigated teacher knowledge in integrating GeoGebra into mathematics classrooms in Turkey and India through capacity-building workshops. In these studies, mathematics teachers were exposed to GeoGebra via training sequences.

After training, the mathematics teachers' perspectives on capacity building and the ultimate use of GeoGebra as a pedagogical tool for augmenting mathematics learning were investigated. Findings indicated that they were enthusiastic about using GeoGebra during their instruction because they perceived it as useful for teaching and learning mathematics. However, it was noted that the mathematics teachers' actual use of GeoGebra significantly differed from their perceptions, in that they struggled while using GeoGebra in authentic classroom contexts [35]. In the case where mathematics teachers utilized TPACK while creating classroom activities, which they planned using GeoGebra, they had difficulties incorporating their pedagogical knowledge into the technology throughout the activity creation processes and further possessed deficiencies regarding their TPACK [27]. Similarly, Açıkgül and Aslaner [2] investigated the effect of using GeoGebra-aided micro-teaching tools and TPACK game trials on prospective mathematics teachers' TPACK efficacy and self-efficacy levels regarding polygons. They noted that although the entire appliance process augmented these teachers' TPACK effectiveness and self-efficacy scores, there was a statistically significant difference between their TCK and TPACK efficacy levels after training. This meant that they had difficulties in selecting the content to teach using technology.

These findings indicate a gap between mathematics teachers' perceptions and knowledge acquisition during training and their implementation of GeoGebra in actual classroom contexts. In other words, although mathematics teachers perceive this interactive software as beneficial, they find it difficult to use it in actual classroom contexts and therefore, there is a need and urgency to not only support them to use GeoGebra as a once-off activity during training but to do so relentlessly through continuous capacity-building programs. This is evidenced in Kartal and Çınar's [26] study in which they investigated pre-service elementary mathematics teachers' TPACK development through a training workshop. They discovered that participants did not manipulate technology effectively and efficiently in their micro-teaching sessions and the first lessons in their student teaching. However, after assessing their first lessons in schools and giving them constructive feedback and continuous support, the teachers enhanced their teaching appreciably. Thus, such findings indicate the need for mathematics teachers to be given more opportunities to teach with technology during their teacher education training or in their classrooms to improve their knowledge. Also, their teaching practices need to be assessed regularly and reflectively to give them support where it is due.

On the contrary, in other studies, such as that of Bhagat et al. [10] in which they developed Indian pre-service mathematics teachers' TPACK to incorporate GeoGebra in teaching secondary school mathematics through a training workshop, scholars revealed positive transforms in teachers' TPACK after being subjected to the GeoGebra training, without any struggle. From this literature, it can be noted that while in some contexts training mathematics teachers to use GeoGebra yields positive results immediately after training them, in other contexts, they continue struggling with its use. This means that the impact of mathematics teachers' capacity building through training workshops varies according to context. Further literature indicates the need for all mathematics teachers to be trained to use GeoGebra before they can ably incorporate it in their teaching and teacher-training contexts. Particularly, in the context of this study, since third-year pre-service mathematics teachers had never had an opportunity, be it in their teacher education training or otherwise, to incorporate GeoGebra in teaching and learning mathematics, it was necessary to expose them to how GeoGebra can be used to teach and learn geometry through a three-week intensive capacity building workshop.

3.2 Effect of using GeoGebra on students' achievement

According to Hillmayr, Ziernwald, Reinhold, Hofer, and Reiss [20], teacher support in integrating technology into mathematics classrooms improves teachers' knowledge and enhances students' mathematical abilities, retention, and achievement. Wassie and Zergaw [51] explored the part that GeoGebra plays in fostering student achievement and teacher effectiveness and maintained that its smart integration in an apt classroom scenario creates a pliable environment, engrosses students, and boosts collaborative learning. Furthermore, Nzaramyimana, Mukandayambaje, Iyamuremye, Hakizumuremyi, and Ukobizaba [37] observed that incorporating GeoGebra in mathematics classrooms can enhance students' gratification at all instruction levels while emboldening the development of skills that are crucial for work and promoting active learning at the same time.

Through quasi-experimental studies, scholars such as Shadaan and Leong [42] and Arbain and Shukor [4] in Malaysia, Zulnaidi, Oktavika, and Hidayat [52], and Simbolon and Siahaan [43] in Indonesia, Mthethwa, Bayaga, Bossé, and Williams [34] in South Africa, Birgin and Topuz [11] in Turkey, and Uwurukundo et al. [49] in Rwanda investigated students' understanding, abilities, retention, and achievement in mathematics using GeoGebra. Using groups of students in their respective countries and at different times and in different years, they all divided them into two, namely, the control and experimental groups. The students in the control groups were taught mathematical concepts such as circles, statistics, functions, and geometry using conventional teaching methods. In contrast, the students in the experimental groups were taught very similar concepts using GeoGebra. In all these studies, the scholars found differences in mean scores of the post-tests between the control and experimental groups to the effect that the students in the experimental groups, who were subjected to the use of GeoGebra, scored higher than those in the control groups who were exposed to conventional teaching methods. These different scholars all found out that incorporating GeoGebra in teaching and learning mathematics enriches students' abilities, understanding, retention, and performance or achievement in mathematics.

From the literature reviewed in this article, it is clear that the efficacy of GeoGebra in teaching and learning mathematics and the impact of teacher support in the form of capacity development in using this interactive software have been studied. However, the use of GeoGebra is a relatively new phenomenon in Uganda. Mathematics teachers have not been exposed to it as part of their teacher training or in-service continuous professional development programs due to its novelty. For this reason, there is a dearth of empirical studies in Uganda that have been documented on the mathematics teachers' capacity-building in using GeoGebra and the effect of using the same on students' achievement. Thus, one might wonder if similar results as those in the reviewed literature would be obtained if mathematics teachers were introduced to using GeoGebra during their teacher training, leading to the instruction of secondary school students with this interactive software. Based on this background, this current study was intended to fill this gap.

4 Method

4.1 Research approach and design

The study employed the quantitative research approach [12], cross-sectional survey [7], and quasi-experiment [6] research designs. A cross-sectional survey was chosen for this study because data about pre-service mathematics teachers' TPACK before and after a capacity-building workshop and secondary school students' achievement in geometry before and after teaching them with GeoGebra and conventional teaching methods were collected and analyzed across a sample population at a specific point in time. The quasi-experiment research design was chosen to evaluate the effect of using GeoGebra on secondary school students' achievement in geometry, an intervention that did not use randomization. The focus was to demonstrate the possible causality between this intervention and the outcome.

4.2 Participants and sampling strategies

Uganda has 11 government universities with a total population of 937 third-year pre-service mathematics teachers. According to Krejcie and Morgan's [29] table of sample size determination, a total sample of 274 pre-service mathematics teachers, as determined from the table, would represent this population size. As such, stratified random sampling was used to determine the number of pre-service mathematics teachers from each of the 11 government universities

who would participate in this study. To get this, the number of pre-service mathematics teachers from each university was divided into 937, the total population, and this quotient was multiplied by 274, the total sample size. It followed then that each university's sample size was selected through simple random sampling [9], whereby each pre-service mathematics teacher was given an unbiased opportunity to participate in the study. Although the total sample size of 274 was arrived at, only 94 consented to voluntarily participate in this study, as most of them had a phobia of using mathematics technologies for the first time.

The 94 participants were then exposed to a three-week intensive workshop, which was facilitated by the researcher and four other experts in ICT in pedagogy, to build their knowledge and capacity to use GeoGebra in teaching and learning geometry, particularly geometric construction and angle properties of geometric figures, at the secondary school level. The other four ICT experts were also proficient in secondary school mathematics. Two weeks after the workshop, the participants had to undergo school practice in secondary schools as part of their teacher education training. During school practice, four participants from the 94 were purposively selected [39] for a quasi-experiment to establish the effect of using GeoGebra in teaching and learning geometry on secondary school students' achievement. The criterion for this sampling was based on three factors. First, for ease of movement for the researcher, the schools in which these pre-service mathematics teachers selected to practice had to be in central Uganda. Second, they should have had well-equipped computer laboratories. Third, the pre-service mathematics teachers should have been allocated Senior Two (S.2) class streams in which geometry is taught. Eventually, while two of the selected teachers were from two private schools, the other two were from two government schools.

Secondary schools that had functional computer laboratories, internet connectivity for downloading GeoGebra software, and provided computer literacy education to students were considered. In each selected school, a non-randomized quasi-experiment was conducted. Two class streams of Senior Two (S.2) students aged 13–15 years participated in this experiment, whereby one S.2 class stream became the control group and the other the experimental group. All class streams were left intact in terms of the number of learners in each and thus, were selected to participate in this study through census sampling [22]. Since the geometry lessons were part of their learning syllabus, all learners consented to participate in the study. Overall, there were four control groups and four experimental groups with 207 and 199 students respectively. In total, 406 S.2 students participated in the quasi-experiment. In each school, the pre-service mathematics teachers taught geometric construction and angle properties of geometric figures to both groups of learners for six weeks. Each group had the same volume of work and working time and the only difference was that different teaching methods were used for each group. That is, the experimental groups were taught using GeoGebra, and conventional teaching methods were used in the control groups. Many Ugandan learners struggle to conceptualize and retain geometric concepts, which ultimately results in poor performance on geometric tasks in national examinations [48]. The use of both GeoGebra and conventional teaching methods was to determine which, if carefully implemented and given sufficient learning time, could improve learners' conceptual understanding and retention of geometric concepts more effectively.

4.3 Data collection tools

Data in this study were collected through self-administered questionnaires (SAQs) and achievement tests. On the first day of the workshop, the 94 pre-service mathematics teachers filled out a SAQ that measured their self-rating on the seven TPACK knowledge constructs: CK, PK, TK, PCK, TPK, TCK, and TPACK. The SAQ was adapted from Batiibwe, Bakkabulindi and Mango [8]. CK, PK, TK, PCK, TPK, TCK, and TPACK had 3, 7, 7, 5, 4, 4, and 6 items respectively. The SAQ was considered reliable and valid based on the original study it was adapted from, so it was not pilot-tested specifically for this study. Instead, its validity and reliability were assessed using the original dataset of this study. The SAQ was valid since when tested, it had a content validity index (CVI) of 0.93 [3]. In addition, the constructs CK, PK, TK, PCK, TPK, TCK, and TPACK had reliability coefficients of 0.913, 0.909, 0.948, 0.917, 0.906, 0.919, and 0.934 respectively and therefore the SAQ was considered reliable. After the workshop, the participants completed the same SAQ to establish if there were any changes in their knowledge as a consequence of the training workshop.

Considering that the 406 secondary school students had some knowledge of geometry from their primary school education, they were given an achievement test before teaching commenced. The test on geometric construction and angle properties of geometric figures had ten multiple-choice questions and ten problem-solving questions. The five facilitators, that is, the author and four other ICT experts who were selected because of their expertise and rich experiences in ICT facilitation, ensured that the test was valid and reliable. The content validity index (CVI) of 0.92 that was calculated on the test items indicated that it was valid. Further, item analysis of the test was carried out by calculating the difficulty and distinctiveness

and/ or reliability/consistency of the test items using Kuder-Richardson Formula 20 (KR-20). Since the calculated value of KR-20 was 0.89, the test was considered reliable. Each multiple-choice question was awarded two marks if correctly answered, while that of problem-solving was similarly awarded four marks. The marks of two and four were awarded based on the order of thinking skills required for the questions. The test was scored out of 60 marks but later converted to the 100 percent scale during analysis. This same achievement test was given to all 406 secondary school students after teaching, and it was scored similarly. However, while the numbering of the questions was altered, the content was left intact.

4.4 Data analysis

Data from the TPACK SAQs and achievement tests were analyzed using descriptive and inferential statistics. In particular, mean scores from the 94 pre-service teachers' pre- and post-TPACK surveys and the 406 S.2 secondary school students' pre- and post-achievement tests for both the four control and four experimental groups were computed. Further, these were compared employing independent samples *t*-tests and paired samples *t*-tests, which were performed utilizing the Statistical Package for Social Sciences Version 21.0 (SPSS 21.0) software. Specifically, the non-parametric *t*-tests were exploited to examine the statistical significance difference in the pre-service mathematics teachers' knowledge and achievement between students in the control and experimental groups before and after the intervention. The independent samples *t*-test was used to compare the means of two independent groups of students in the experimental and control groups. The paired samples *t*-test was selected to compare the pre-service teachers' prior knowledge with their knowledge after the training, meaning that this difference was within the same subjects. To measure the difference between two groups' means, Cohen's *d* standardized effect size was computed using SPSS. This effect size (*d*) was used to tell how meaningful the difference between the groups' means was. Cohen's *d*=0.20, 0.50, and 0.80 were used to interpret observed effect sizes as small, medium, or large, respectively [30].

4.5 Ethics

The purpose and objectives of the study were explained to all the participants. All the 94 third-year pre-service mathematics teachers and 406 S.2 secondary school students consented to participate in this study by signing consent and assent forms. Before the assent of students, permission was sought from the schools' head teachers and directors of studies for the researcher to access the schools and later on the classrooms. In addition, all participants voluntarily took part in this study and therefore were free to leave at any one point during the study. The participants did not write their names on the SAQs and tests, making them anonymous and confidential solely for research purposes.

5 The implementation process of the study

To visualize the implementation process of the study, a flowchart was drawn. Thus, Fig. 2 shows the flowchart of the implementation process of the study.

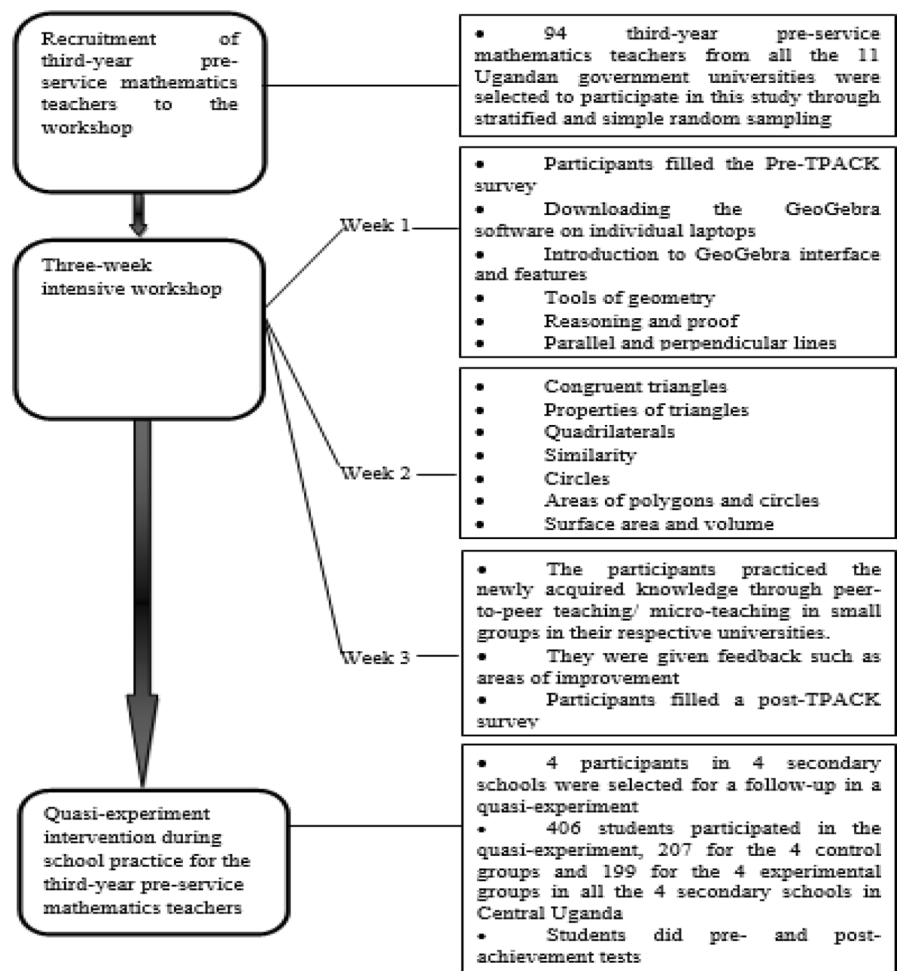
During the third week of the workshop, the 94 third-year pre-service mathematics teachers from Uganda's government universities engaged in peer-to-peer instruction through microteaching. This was aimed at providing practical experience with GeoGebra to those who had not attended the training session at their respective universities. Furthermore, this would give the study participants a chance to practice their newly acquired knowledge among their peers before they could take part in their school practice. During micro-teaching the five facilitators observed the participants' teaching, gave constructive feedback, and supported them to enhance their teaching practices. Before micro-teaching, the participants were given a week to prepare lesson plans for four lessons in geometry using their training materials. Each participant worked with 9–10 peers for two weeks. Within each week, two lessons were conducted and each lesson lasted for two hours. Finally, the 843 remaining third-year pre-service mathematics teachers in all 11 government universities, who did not participate in the workshop, attended the micro-teaching lessons.

6 Findings

6.1 The impact of capacity-building on third-year pre-service mathematics teachers' TPACK

Capacity building is the process of imparting knowledge and developing skills while optimizing the advantages of participation and knowledge sharing. In the context of this study, capacity building was necessary to assist pre-service

Fig. 2 The implementation process of the study (designed by the researcher)



mathematics teachers in developing the specialized knowledge, TPACK, required to incorporate technology, such as GeoGebra, into the teaching and learning process. The pre-service mathematics teachers rated their seven knowledge constructs, namely, CK, PK, TK, PCK, TPK, TCK, and TPACK, before and after the training using a five-point Likert scale that ranged from 1 = Strongly Disagree (SD), 2 = Disagree (D), 3 = Undecided (U), 4 = Agree (A), and 5 = Strongly Agree (SA). To ascertain whether there was a difference in the teachers’ ratings of their knowledge, the mean scores on each knowledge construct before and after the training were calculated and then later contrasted using a paired samples *t*-test at the 0.05 significance level. Table 1 shows these results. Although the paired samples *t*-test assumes that the differences between pairs are normally distributed, a Shapiro–Wilk test was further done to confirm normality. Since the *p*-value was greater than 0.05, the significance level, and the test statistic, *W*, equal to 0.961, the data were normally distributed. The paired samples *t*-test did not assume that the variances of both samples were equal.

Table 1 reveals that the pre-service teachers’ mean score on CK before the training was 4.12 and that after the training was 4.46. While the value 4.12 corresponds to code 4 of the scale, that of 4.46 corresponds to code 5, which means agree and strongly agree on the scale. This indicates that the pre-service mathematics teachers rated themselves high on CK before and after the training. Further, the *p*-value on CK was larger than the significance value of 0.05 ($p > \alpha$), giving rise to a large *t*-statistic of -1.967 . Thus, there was no statistically significant difference between the pre-service mathematics teachers’ CK mean scores before and after the training. However, the effect size of $d = 0.7$ indicates that their CK before and after training had a moderate difference although not statistically significant. A similar trend happened with the pre-service mathematics teachers’ PK and PCK whereby the mean scores on their PK and PCK before and after the training, 4.06 and 4.23 and 3.81 and 4.37, respectively, corresponded to code 4 of the scale.

The implication was that since code 4 of the scale compared to agree, the pre-service mathematics teachers still rated themselves high on PK and PCK. Furthermore, the *p*-values on PK (0.511) and PCK (0.446) were all larger than the significance value of 0.05, also giving rise to large *t*-statistics of -1.287 and -1.371 respectively, which means that there

Table 1 Results of the Paired Samples *t*-Test on Third-Year Pre-service Teachers' TPACK before and after the Training

Knowledge construct	Overall mean before training	Overall mean after training	Overall SD before training	Overall SD after training	<i>t</i> -statistic	<i>p</i> -value	Effect Size (<i>d</i>)
CK	4.12	4.46	0.481	0.213	-1.967	0.354	0.7
PK	4.06	4.23	0.722	0.432	-1.278	0.511	0.2
TK	1.96	3.68	2.11	1.962	2.974	0.033	0.8
PCK	3.81	4.37	1.017	0.531	-1.371	0.446	0.6
TPK	2.42	3.79	1.754	1.008	3.784	0.028	0.8
TCK	2.09	3.61	1.239	0.974	1.998	0.041	1.2
TPACK	2.15	3.83	1.695	0.977	4.192	0.019	1.0

The significant value was at $\alpha=0.05$, and the *t*-value was significant at $p < 0.05$

was no statistically significant difference between the pre-service mathematics teachers' PK and PCK before and after the training. In addition, while PK had a small effect size of $d=0.2$, PCK had a medium one of $d=0.6$. This means that the pre-service mathematics teachers' PK before and after training had a negligible difference, while their PCK had a moderate difference, although in both cases not statistically significant.

Regarding TK, the pre-service mathematics teachers' mean scores before and after the training were 1.96 and 3.68, respectively. These scores correspond to codes 2 and 4 on the scale. This means that before the training, the pre-service mathematics teachers rated themselves low on TK, and after the training, they rated themselves high, meaning that the training positively impacted their TK. The *p*-value on TK scores was 0.033, which gave rise to a large *t*-statistic equal to 2.974. The *p*-value is less than the significance value of 0.05 ($p < \alpha$), meaning that there was a statistically significant difference between the pre-service mathematics teachers' TK before and after the training. These results are consistent with all the other technological knowledge constructs, namely TPK, TCK, and TPACK. The pre-service mathematics teachers' mean scores on TPK, TCK, and TPACK before the training were 2.42, 2.09, and 2.15, respectively, and those after the training were 3.79, 3.61, and 3.83, respectively.

All the mean scores on TPK, TCK, and TPACK before the training corresponded to code 2 of the scale, meaning that teachers rated themselves low on these. However, after the training, the teachers ranked themselves high since all their mean scores corresponded to code 4 of the scale. Furthermore, the *p*-values on TPK, TCK, and TPACK were 0.028, 0.041, and 0.019, which gave rise to large *t*-statistics of 3.784, 1.998, and 4.192, respectively. Given that all these *p*-values were less than the significance value of 0.05, there were statistically significant differences between the pre-service mathematics teachers' TPK, TCK, and TPACK before and after the training. The training reportedly impacted the pre-service mathematics teachers' TK, TPK, TCK, and TPACK, as there was a positive change in their self-rating on these constructs after the training. In addition, the differences between their TK, TPK, TCK, and TPACK before and after training had effect sizes of $d=0.8$, 0.8, 1.2, and 1.0 respectively, showing that they were substantial.

6.2 Effect of using GeoGebra on secondary school students' achievement in geometry

To ascertain whether there existed any significant difference between the pre-test mean scores of secondary school students in both the control and experimental groups, an independent samples *t*-test was performed. Table 2 shows these results. In addition, a Shapiro–Wilk test was performed to verify normality. The results showed that the data were normally distributed because the test statistic, *W*, equaled 0.965 and the *p*-value (0.837) was higher than 0.05, the significance level. The homogeneity assumption of the variance was tested using Levene's test. The *p*-value for Levene's test was 0.567, which was greater than 0.05, so the variances were not significantly different from each other.

Table 2 discloses that the control group acquired a mean score of 55.8 while the experimental group attained 56.2 in the pre-test. The mean score difference between the groups was 0.40, with a *t*-statistic value of -1.267. Nevertheless, the *p*-value was 0.126, which was larger than the significance value of 0.05 ($p > 0.05$), demonstrating that the difference in the mean scores of the two groups was insignificant. In addition, an effect size of Cohen's $d=0.1$ further confirms the insignificant difference [30]. This outcome revealed that secondary school students in the control and experimental groups were analogous in aptitudes before administering the intervention.

To establish whether there existed any significant differences between the post-test mean scores of secondary school students in both the control and experimental groups, an independent samples *t*-test was carried out. Table 3

Table 2 Results of the independent samples *t*-test on secondary school students' pre-test of both groups

Student group	Sample size	Mean score	SD	<i>t</i> -statistic	<i>p</i> -value	Effect Size (<i>d</i>)
Experimental	199	56.2	4.04	-1.267	0.126	0.1
Control	207	55.8	6.25			

The *t*-statistic was significant at $p < 0.05$

shows the results. Further, a Shapiro–Wilk test was performed to verify normality. The results showed that the data were normally distributed because the test statistic, *W*, equaled 0.867 and the *p*-value (0.739) was higher than 0.05, the significance level. The homogeneity assumption of the variance was tested using Levene's test. The *p*-value for Levene's test was 0.613, which was greater than 0.05, so the variances were not significantly different from each other.

Table 3 exhibits that the control group acquired a mean score of 69.4 while the experimental group got 87.6 in the post-test. The mean score difference between the groups was 18.20, with a *t*-statistic value of 4.129. Still, the *p*-value of 0.001 that was obtained was less than the significance value of 0.05 ($p < 0.05$), suggesting that the difference in the mean scores of the two groups was significant. In addition, an effect size of Cohen's $d = 5.7$ further confirms the significant difference [30]. This outcome exemplified that the secondary school students in the experimental groups achieved higher in the geometry test using GeoGebra than those in the control groups whose teachers used the conventional teaching methods.

A paired samples *t*-test was performed to contrast the pre-and post-test scores for the experimental and control groups. The results are illustrated in Table 4. A Shapiro–Wilk test was done to confirm normality. Since its *p*-value of 0.324 was greater than 0.05, the significance level, and the test statistic, *W*, equal to 0.934, the data were normally distributed. The paired samples *t*-test did not assume that the variances of both samples were equal.

Table 4 demonstrates that the mean score difference between the pre-test and post-test of the experimental group was 68.7, as contrasted to the control group with 52.2. For the experimental group, the *t*-statistic value found was 5.94, and the *p*-value was less than the significance value of 0.05 ($p < 0.05$), signifying that the difference between the pre-and post-test scores was significant. For the control group, the *t*-statistic attained was 16.43, and the *p*-value found was less than the significance value of 0.05 ($p < 0.05$), suggesting the difference between the pre-and post-test scores was significant. Thus, this confirms that there was a substantial enhancement in the scores of secondary school students in both the experimental and control groups. It can easily be observed that secondary school students benefited from both teaching and learning modes. However, those in the experimental groups showed a greater average improvement in scores compared to the control groups.

Table 3 Results of the independent samples *t*-test on secondary school students' post-test of both groups

Student group	Sample size	Mean score	SD	<i>t</i> -statistic	<i>p</i> -value	Effect Size
Experimental	199	87.6	3.17	4.129	0.001*	5.7
Control	207	69.4	4.28			

The *t*-statistic value was significant (*) at $p < 0.05$

Table 4 Results of the paired samples *t*-Test

Pair		Mean score difference	SD	<i>t</i> -statistic	Sig (2-tailed)
Pair 1	Experimental post-test score-pre-test score	68.7	4.89	5.94	0.001
Pair 2	Control post-test score- pre-test score	52.2	6.44	16.43	0.001

The *t*-statistic significant at $p < 0.05$

7 Discussion

Capacity building exposed the third-year pre-service mathematics teachers' knowledge of using GeoGebra in teaching and learning geometry. Specifically, they were confident about their CK, PK, and PCK, which they self-rated high before and after the training. However, while they rated themselves low on technological knowledge constructs, namely TK, TPK, TCK, and TPACK before, they rated themselves high after the training and would surmise that their knowledge increased. These findings corroborate other studies [2, 10, 26, 36] which were done in Turkey, India, Turkey, and Indonesia, respectively, to determine the impact of capacity building through training teachers on how to use different technological tools, such as GeoGebra [4, 11, 14, 16] in teaching and learning mathematics in Malaysia, Turkey, Argentina, and the United Kingdom, among other countries. Thus, training mathematics teachers on how to exploit mathematics-specific technology is imperative. The Ugandan Ministry of Education and Sports (MoES) provides professional development training for science and mathematics teachers every school holiday, including the effective use of technology [31]. However, this is significantly affected by poor internet connection, low bandwidth, frequent power shortages, and high data bundle costs [19]. Additionally, the initial mathematics teacher education at universities in Uganda neither offers courses that provide pre-service mathematics teachers with hands-on experiences with technology for instructional use nor arrange for technology-rich environments in which they are trained [8, 38] during their teacher preparation.

This study has demonstrated that GeoGebra, a subject-specific software, can be utilized as an enabler in the teaching and learning of mathematics, such as in geometry, since there was a significant improvement in the experimental group students' conceptual understanding and later on achievement, as contrasted to the control group students. Moreover, given that it is a freely downloadable software that requires no internet connection for its subsequent use, it can be successfully incorporated within these professional development courses MoES conducts during the school holiday and later in mathematics teacher training. During the training, pre-service mathematics teachers freely interacted for support, and the software offered tutorials beyond the training period. Similar to many others [4, 11, 14, 34, 42, 43, 49, 52], this study found that there was a significant enhancement in achievement in geometry among students in the experimental groups who were instructed using GeoGebra. With the availability of subject-specific technologies, like GeoGebra, teachers can create graphical depictions of mathematical notions and thus present complex concepts fluently and conspicuously. Further, Zulnaidi et al. [52] maintained that GeoGebra could represent mathematical notions and processes suitably using illustrations and drawings, which substantially support students in understanding ideas and techniques concerning mathematics, such as geometry and algebra. Moreover, GeoGebra is comprehensible and alleviates the teachers' encumbrance of explicating abstract mathematical notions. Furthermore, utilizing GeoGebra while teaching mathematics makes the students' learning process progressively dynamic, and most importantly, it permits effective collaboration between teachers and students.

Previous research has demonstrated the benefits of GeoGebra in settings other than Uganda. However, prior to this investigation, it was unclear whether the results would be sufficient in Uganda, where challenges with low bandwidth and unreliable internet connectivity coexist with insufficient resources. According to the study's findings, GeoGebra has the same effects anywhere it is downloaded, regardless of geographical location. This suggests that policymakers, such as national curriculum centers, should consider including the use of GeoGebra in teaching methods outlined in syllabi manuals. Additionally, it is important to integrate GeoGebra into ongoing professional development programs, and teachers should be trained in how to use this software. They should also be encouraged to adopt its use in their daily teaching practice. The study's findings add to the limited research on GeoGebra in sub-Saharan Africa, where it is a relatively new phenomenon.

8 Conclusion

In this study, the GeoGebra interactive software has attested to be an efficient tool in augmenting teaching and learning geometry. Notably, the students' hands-on learning experience unveiled a positive influence in empowering them to comprehend geometry concepts better. Therefore, secondary schools and mathematics teacher training institutions in Uganda should consider downloading and using the GeoGebra interactive software to enhance mathematics students' abilities to grasp the subject's precepts. As a qualification, particular emphasis must be placed

on mathematics teachers through capacity building for its eventual practical application in classrooms. Although positive results were obtained in this study, it was limited to only government universities. Therefore, findings cannot be generalized to private universities. Therefore, further studies should be conducted in private universities to establish if these findings would be consistent. Besides, although 94 pre-service mathematics teachers were the sample in this study, it is such a small number compared to 937, the total number of pre-service mathematics teachers in government universities in Uganda, and therefore further studies involving more pre-service mathematics teachers could yield different results. It is also noted that the pre-service mathematics teachers were rating themselves on their knowledge domains. One cannot be sure of how authentic the rating could be and therefore cannot be very sure about the impact of the training workshop. The results could be different if someone independent was assessing or rating them. Furthermore, other theories such as the conversation and activity theories can be used by other researchers to report what happens during mathematics teachers' capacity building in using GeoGebra and later on the effect it has on students' achievement.

Author contribution M.S.K.B wrote the entire manuscript and reviewed it.

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Data availability All data generated and analyzed during this study are included in this published article.

Declarations

Ethics approval and consent to participate The Makerere University School of Social Sciences Research Ethics Committee exempted this study from research clearance. All the 94 pre-service mathematics teachers signed consent forms to confirm their voluntary participation in this study and the publication of the study's findings. Given that mathematics learning is compulsory for all S.2 students in Uganda and that the schools from which the 406 S.2 student participants came were boarding schools, informed assent was obtained from their head teachers and directors of studies, who at the time they were in boarding schools, were their legal guardians. These signed the assent forms confirming their students' participation in this study and the publication of the study's findings.

Competing interests The authors declare no competing interests.

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