Geoid Determination In Uganda: Current Status

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ABSTRACT

Many professionals e.g. surveyors, engineers and GIS specialists are increasingly using Global Positioning System (GPS) or some other Global Navigation Satellite Systems (GNSS) for positioning and navigation. One of the greatest advantages of GPS is its ability to provide three-dimensional coordinates (latitude, longitude and height) anywhere in the world, any time irrespective of the weather. The GPS latitude and longitude can easily be transformed from the WGS84 reference system to a local reference (e.g. Arc 1960). However the GPS-determined heights, i.e. ellipsoidal heights, are geometrical heights which have no physical meaning and therefore cannot be used in surveying and engineering projects. Their conversion to more meaningful orthometric heights require knowledge of the geoidal undulations, which can be determined from high resolution geoid models. Its absence in Uganda means that the full potential of GPS cannot be fully realized. This paper gives an overview of the need for an accurate geoid model in Uganda, the current status of the geodetic network in Uganda and different methods of geoid determination. Pending further investigation, preliminary findings indicate that in Uganda, the EGM2008 is the best geoid modal for GPS/leveling projects.

Keywords: Geoid model; Global Positioning System; orthometric heights.

1.0 INTRODUCTION

The geoid is an equipotential surface, i.e. a level surface to which the direction of gravity is perpendicular. By definition, the geoid corresponds to the surface that approximates Mean Sea Level (MSL) globally (Veronneau *et al.*, 2006). However, MSL is not an equilibrium surface in the earth's gravity field due to ocean currents and other quasi-stationary effects. The Permanent Sea Surface Topography (SST) - the difference between the geoid and the actual MSL ranges globally from -1.8 m to +1.2 m (Veronneau *et al.*, 2006). Consequently, at the 'cm' accuracy level, a regional geoid model is defined as the level surface which optimally fits MSL at a selected set of tide gauges used for defining the vertical datum of a national or continental height system (Torge, 2001).

The geoid is the reference surface for orthometric heights. Orthometric heights are traditionally determined using spirit leveling techniques. This is a very expensive and tedious venture especially when one considers that for a country like Uganda the reference tide gauge is located 1,800km away at Mombasa port in Kenya. This makes height determination using GPS a very attractive alternative. However, the GPS derived heights are ellipsoidal heights whose reference surface is the WGS84 ellipsoid and not the geoid. The ellipsoidal heights can nonetheless be transformed into orthometric heights using the simple geometrical relationship:

$$h = H + N \qquad (1)$$

where *h, H* and *N* are ellipsoidal, orthometric and geoidal heights respectively. From Eq. (1) orthometric height (H) can be determined provided h is derived from GPS measurements and N from a precise gravimetric geoidal model. The determination of orthometric heights (H) represents the primary application of the geoid model. Some of the other applications of an accurate geoid model include determination of a vertical datum, unification of vertical reference systems, determination of the Earth's gravitation field and geodynamic applications as discussed in sections 1.1 -1.4.

1.1 Vertical reference datum

A vertical reference frame or datum forms the basis for all development projects in which heights are needed. All projects involving the impounding, transport and distribution of water are critically dependent upon the appropriate vertical reference frame being used (Merry, 2003). The geoid is an ideal datum for heights since it represents a continuous surface known everywhere across the entire country (Veronneau *et al.*, 2006). This guarantees the continuity of heights across borders and coastline zones (Vaníček, 2009). This has application in the demarcation of the country boundaries over water bodies (Vaníček, 2009).

1.2 Earth's gravity field

The Earth's gravity field contains the most direct information about the mass density distribution within the Earth (Vaníček, 2009). It therefore forms the basis for the exploration of oil, gas and other underground minerals. In this regard geoid determination forms the basis of mineral exploration. Accurate gravity data also has application in earth sciences e.g. in oceanography and satellite dynamics for the determination of satellite orbits.

1.3 Geodynamical applications

The geoid is very important in the modeling of geodynamical phenomena like polar motion, Earth rotation and crustal deformation. It is also useful in the interpretation of precursors to geo-hazards such as post-glacial rebound, earthquakes, volcanoes, landslides, tsunamis and their mitigation (Ulotu, 2009).

1.4 Unification of vertical reference datums

Because of differences in SST at tide gauge sites and differences in measuring techniques, national vertical datums of many African countries differ from each other. This leads to delays in the implementation of regional projects in areas such as transportation, communication, water and electricity reticulation grids and other projects that require heights. Determination of a precise gravimetric geoid represents a significant step in eliminating these differences as it forms the basis for the determination of regional geoid models.

2.0 STATUS OF THE GEODETIC NETWORK IN UGANDA

Geodetic networks consist of permanently monumented control points that comprise the framework for a national reference frame from which surveying and mapping work in the country is carried out. Geodetic networks play an important role in the determination of an accurate geoid model as the orthometric heights are determined on the basis of the vertical network of the country. In many countries in Africa, Uganda inclusive, separate horizontal and vertical networks were established by the 1960's. The horizontal network would be tied to a chosen ellipsoid and the vertical network referenced to MSL through measurements to one or more tide gauges.

2.1 Horizontal network

By 1960 most of Uganda's geodetic control network was established. The triangulation network consisted of primary, secondary and tertiary control points distributed throughout the country. The network was necessitated by the work of the Boundary Commissions of the British Colonial Administration to define the Congo-Uganda and Tanganyika-Uganda boundaries. This was in addition to the measurement of the arc of the 30th meridian and the Mailo surveys of Buganda (IGN, 2003). The computation of this network was originally carried out on the Clarke 1858 ellipsoid using the triangulation chain along the arc of the 30th meridian as control (Okia & Kitaka, 2003). In 1960, a re-computation of the main triangulation network was carried out on the Clarke 1880 ellipsoid, whose semi-major axis (a) is 6378249.145 m and reciprocal flattening (1/f) is 293.4663, leading to the naming of the Ugandan datum as the '1960 arc datum' (Okia and Kitaka, 2003). Overall, a total of 1,730 control points were established consisting of 130 primary control points spaced between 30 km to 80 km, 650 secondary control points spaced between 20 km to 50 km and 950 tertiary control points spaced between 5 km to 10 km (Okia and Kitaka, 2003). However, most of the markers of these controls were destroyed during the period of political turmoil i.e. 1970's to 1980's. Currently, it is estimated that of the original control points, there only exists approximately 50 cross-cuts on hard rock including 14 primary, 27 secondary and 8 tertiary control points (IGN, 2003).

2.2 Vertical network

Before the Second World War, the British Directorate of Overseas Surveys (DOS) carried out precise differential leveling from the Kenyan coast (Mombasa) to Uganda. As the reference sea level for this exercise was in Mombasa, the datum was called MSL Mombasa. Another differential leveling project was commissioned from Egypt to Uganda which passed through Sudan. In this case the reference sea level was obtained from the harbor of Alexandria, and new calculations for the heights of all the benchmarks were carried out in the so-called New Khartoum datum (the fundamental point of which is given at 363.082 meters above MSL at Alexandria). This was consequently tied to the New MSL Alexandria datum (IGN, 2003). By 1970 the First order network was completed and a block adjustment was carried out by the Surveys and Mapping Department in Uganda using the observation equation method producing values referring to MSL Alexandria with a standard error of 0.00115 feet per unit weight (IGN, 2003). The difference between the new datum and the old one was calculated as -0.055 foot (MSL Mombasa is 0.055 feet lower than MSL Alexandria at the Khartoum gauge). Therefore the heights in Uganda are based on only one connection to the Egyptian Benchmark BM 9029 which is related to MSL Alexandria. However, check connections with Egyptian BM 927 have revealed a disagreement of -0.1497 feet (IGN, 2003). Overall, a total of 3033 benchmarks consisting of 51 fundamental benchmarks and 1015 town benchmarks were listed in 1972 in Uganda. The heights of these benchmarks were computed in the New Khartoum vertical datum, with respect to the MSL in the Red Sea. However, there is little information about the current state of these benchmarks although most of the benchmarks that were sited on buildings are presumed to still exist.

3.0 SOME METHODS OF GEOID DETERMINATION

3.1 Gravimetric methods

The geoid height (N) can be determined from gravity data using the Stokes formula.

$$N = \frac{R}{4\pi\gamma_m} \iint_\sigma S(\psi)\Delta g d\sigma \qquad (2)$$

where R is the mean Earth's radius, ψ is the geocentric angle, Δg is the gravity anomaly, $d\sigma$ is an infinitesimal surface element of the unit sphere σ, γ_m is normal gravity on the reference ellipsoid and $S(\psi)$ is the Stokes function. The determination of *N* using equation (2) requires tedious numerical integration of gravity anomalies, which must be carried out over the entire globe. Consequently, several modifications of Stokes' formula, which combine terrestrial gravity anomalies and long-wavelength information from a Global Geopotential model (GGM) have been devised by several researchers (e.g. Molodensky, 1962; Sjöberg, 1984). Among the commonly used methods are:

- Remove-Compute-restore (RCR) method developed at the University of Copenhagen, Denmark

- Stokes-Helmert approach, developed at the University of New Brunswick, Canada
- Least Squares Modification of Stokes' formula with additive corrections (LSMSA), developed at the Royal Institute of Technology (KTH), Sweden.

However, the gravimetric method suffers from the following drawbacks (Merry, 2008);

- It is computationally intensive and mathematically complex
- The precision of the resultant geoid depends heavily on the quality and quantity of the terrestrial gravity data used in the computation.
- The gravimetric geoid is susceptible to biases and tilts due to errors in satellite orbit modeling and to gaps in terrestrial gravity data sets.

Of the listed gravimetric models, the LSMSA is gaining prominence since winning the geoid modeling competition at the International Hotine-Marussi symposium in 2009 (Ågren *et al.*, 2009a). The LSMSA technique was also the preferred approach (over the RCR technique) by the National Land Survey of Sweden in developing its new national geoid model (Ågren et al., 2009b). The LSMSA method was developed at the Royal Institute of Technology (KTH) Division of Geodesy by Sjöberg (1984a), (1984b), (1991), (2003a) and (2003b). The method minimizes the expected global Mean Square Error of the estimated geoid height. Hence, in contrast to most other methods of modifying Stokes' formula, which only strive for reducing the truncation error, the LSMSA matches the errors of truncation, gravity anomaly and the GGM in a least squares sense. Another unique feature of the method is that topographic, atmospheric and ellipsoidal corrections are applied separately as additive corrections to the preliminary estimated geoid heights. The method has been used in determination of precise gravimetric geoids in developing countries with sparse terrestrial gravity data (Nsombo, 1996; Hunegnaw, 2001; Kiamehr, 2006; Abdalla, 2009, Ulotu, 2009)

3.2 Geometric

The geometric method of geoid determination is based on a re-arrangement of equation (1) such that

$$N = h - H \qquad (3)$$

A geoid model over a region of interest is determined by making GPS measurements on a number of control points whose orthometric heights are known over the region of interest and using modeling techniques like polynomials, splines and kriging. However the resultant model will only be valid over the area covered by the control points. The challenge in places however arises when there isn't sufficient number of control points over the area of interest.

3.3 Combined method

The combined method attempts to combine the strengths and minimize the drawbacks of both the gravimetric and geometrical methods. This procedure first determines a gravimetric geoid and then tests its accuracy and precision by making GPS measurements on a number of control points whose orthometric heights are precisely known. Finally, the gravimetic geoid model is improved by the discrete GPS/leveling derived geoid heights in an optimum way to a combined geoid model. It is proposed that this be the preferred method for the determination of a geoid model for Uganda.

4.0 PREVIOUS GEOID MODELS IN UGANDA

4.1 A High Resolution Gravimetric Geoid of the Eastern African region

This is a regional geoid determined for Kenya, Uganda and part of Tanzania whose boundaries are at latitude $-5° \leq \Phi \leq 5°$ and longitude $29°E \leq \lambda \leq 42°E$ (Gachari and Olliver, 1998). It was computed using the following datasets:

- The coefficients of the Ohio State University 1991 (OSU91A) geopotential model complete to degree and order 360.

- A set of 2.5′x2.5′ 19,106 mean free-air gravity anomalies, Bouguer anomalies and heights compiled by the University of Leeds.
- A set of marine gravity observations from the University of Oxford.
- Sea surface altimeter heights from Seasat, Geosat/ERM, ERS-1 and TOPEX/Poseidon
- A 1- km resolution Digital Terrain Model (DTM) covering a large area of Kenya from Leicester University
- The TerrainBase 5′ DTM obtained from the US National Geophysical Data centre.

This method is based on the RCR technique. The computation process consisted of removing the long-wavelength components as given by the OSU91A geopotential model from the free-air gravity anomalies and using the residual anomalies on 34,272 data grid points to compute the geoid heights using the one-dimensional Fast Fourier Transform. The residual geoid heights together with the indirect effect were then added to the OSU91A geoid heights to give the final gravimetric geoid at a contour interval of 0.5 meter. For independent verification, gravimetric geoid heights were computed on 25 points with Doppler observations. Differences were then determined between the Doppler geoid heights and the computed gravimetric geoid heights. From this comparison, it was concluded that the geoid was accurate to about 10 cm in Western Kenya and 20 cm elsewhere. Two major deficiencies of this model were as follows:

- No Doppler points were found in Uganda. Hence the accuracy of the geoid could not be independently verified in Uganda.
- The accuracies of the Doppler observations (for which heights were determined using trigonometric heighting) are only between 1 to 3m meaning that the computed differences between the Doppler geoid heights and the gravimetric geoid heights are only accurate to the same level.

4.2 African Geoid project Geoid model

The African Geoid Project (AGP) is an attempt to produce a uniform precise geoid model for Africa (Merry, 2003). It was initially established as a project of the Committee for Developing Countries of the International Association of Geodesy (IAG). In July 2003, the project was taken over as a project of Commission II (Gravity Field Commission) of the IAG. A preliminary geoidal model for Africa was computed in 2003 (Merry *et al.*, 2003). In this model, the gaps in the 5' terrestrial gravity data set were filled using the EGM96 geopotential model (Lemoine *et al.*, 1996), and the geoid was computed in the following steps (Merry, 2003);

- The long wavelength component of the height anomalies (quasi-geoid) was computed using the EGM96 geopotential model
- The short wavelength component was computed using reduced gravity anomalies in a two-dimensional convolution representation of the Stokes' integration
- The terrain effect (Molodensky term) was computed using the GLOBE (Hastings and Dunbar, 1998) digital elevation model (DEM)
- The height anomalies were converted to geoidal heights using Rapp's (1997) spherical harmonic representation of the separation between the two surfaces.

The validation of the resultant geoid model AGP2003 with GPS/leveling data from Algeria, Egypt and South Africa showed biases ranging from -17cm to +124cm with standard deviations ranging from 9 to 80 cm. The potential sources of these biases as noted by Merry (2003) include:

- Errors in the long wavelength components of the EGM96 geopotential model
- Differences in the GPS reference frame used
- Biases in the vertical datums used in the different countries
- Cumulative systematic errors in the leveling networks.

However this model has not been independently tested in Uganda or any of the neighboring countries. In addition to the sparse terrestrial gravity data in Uganda, the accuracy of the AGP2003 cannot be expected to be better than in any of the above three countries.

4.3 EGM 2008

The Earth Geopotential Model 2008 (EGM2008) is the latest version of a series of geopotential models developed by the National Geospatial-Intelligence Agency (Pavlis *et al.*, 2008). It incorporates harmonic coefficients derived from the GRACE satellite mission, marine gravity anomalies derived from satellite altimetry, and a comprehensive set of terrestrial gravity anomalies (Merry, 2009). This model was compared directly with an updated version of the African Geoid Project model (AGP, 2007). The results show a mean difference of +0.02 m with a standard deviation of 0.73 m. As with all the other models discussed, there has not been any independent evaluation of this model in Uganda hence its accuracy can only be assumed to be similar to that of the entire continent.

5.0 CONCLUSIONS

Clearly the time for the determination of a precise gravimetric geoid model for Uganda is long overdue. The geoid is not only important in engineering but is also useful in a number of economic activities including mapping, dredging, and mitigation of natural hazards, transportation and navigation. The authors are working on its determination and should be able to produce a preliminary geoid in two years time. However, before this can be realized, the EGM2008 appears to be the best model that can be used in Uganda for GPS/leveling projects although it has to be used with caution.

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