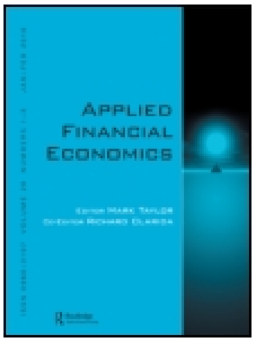

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Predicting BRICS stock returns using ARFIMA models

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This article examines the existence of long memory in daily stock market returns from Brazil, Russia, India, China and South Africa (BRICS) countries and also attempts to shed light on the efficacy of autoregressive fractionally integrated moving average (ARFIMA) models in predicting stock returns. We present evidence which suggests that ARFIMA models estimated using a variety of estimation procedures yield better forecasting results than the non-ARFIMA (AR, MA, ARMA and GARCH) models with regard to prediction of stock returns. These findings hold consistently for the different countries whose economies differ in size, nature and sophistication.

Keywords: fractional integration; long memory; stock returns; long-horizon prediction; ARFIMA; BRICS

JEL Classification: C15; C22; C53

I. Introduction

The recent global financial crisis has rekindled interest in predicting the path of leading economic indicators including asset prices. There is evidence that asset prices, including stock prices, help in predicting output and inflation by acting as leading indicators (Stock and Watson, 2003; Gupta and Hartley, 2013). Further, there are major (asymmetric) spillovers from the stock markets to the real sector of the economy (Lettau and Ludvigson, 2001, 2004; Lettau *et al.*, 2002; Apergis and Miller, 2006; Rapach and Strauss, 2006 amongst others). Asset price bubbles have potential negative effects on the economy. The departure of asset prices from fundamentals can lead to inappropriate investments that decrease the efficiency of the economy (Mishkin, 2007). Hence, the need for accurate predictions of stock returns cannot be overemphasized. The benefits of such forecasts include paving the path for relevant policy decisions in advance, and providing important information for policymakers to appropriately design policies to avoid the impending crisis (Gupta and Modise, 2012).

Stock markets in emerging countries have become an important source for global portfolio diversification. However, there are challenges with regard to predicting stock returns of emerging stock market returns. Emerging markets are generally characterized by lower levels of liquidity and at the same time by a higher volatility than developed financial markets (Barkoulas *et al.*, 2000; Kasman *et al.*, 2009). High volatility in these markets is often marked by frequent and erratic changes, which are usually driven by various local events (such as political developments) rather than by the events of global importance (Bekaert and Harvey, 1997; Aggarwal *et al.*, 1999). These different features may contribute to different dynamics underlying the returns and volatility, making these markets an interesting sphere of research. Understanding of the dynamic behaviour of stock returns in these markets is crucial for portfolio managers, policymakers and researchers. Therefore, the current article focuses on the prediction of stock returns for a group of

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emerging market economies, Brazil, Russia, India, China and South Africa (BRICS).

Financial variables in general and stock market returns in particular do often exhibit long memory and this has become the subject of research since the seminal contributions of Granger (1980) and Granger and Joyeux (1980). The presence of long memory in stock returns and volatility implies that there exists a dependency between distant observations. From a statistical perspective, long memory in these series is associated with a high autocorrelation function, which decays hyperbolically and eventually dies out. Conversely, if correlations between distant observations become negligible, the series is said to exhibit short memory and possesses exponentially decaying summable correlations. Hence, the impacts of a shock on the long-memory processes persist over a long period of time compared to the short-memory processes (Kasman and Torun, 2007). The major economic ramification for the presence of long memory is the contradiction of the weak-form of market efficiency of Fama (1970) which allows investors and portfolio managers to make predictions and to construct speculative strategies. The price of an asset determined in an efficient market is assumed to follow a martingale process in which the current price change is unaffected by its previous value. By implication, the process should have no memory at all. This implies the absence of exploitable excess profit opportunities. However, when return series exhibit long memory, it indicates that observed returns are not independent over time. If returns are not independent, past returns can help predict future returns, thereby violating the market efficiency hypothesis. Consequently, pricing financial assets with martingale methods may not be appropriate if the underlying continuous stochastic process exhibits long memory. Investigating the long-memory property is critical for derivative market players, risk managers and asset allocation decisions makers.

The existence of long memory in financial series has been widely studied. There are also a number of studies that have examined this for the emerging markets' stock returns and volatility. Recent studies include Wright (2001), Kiliĉ (2004), Vougas (2004), Bellalah *et al.* (2005), Assaf and Cavalcante (2005), Cajueiro and Tabak (2005), Bhardwaj and Swanson (2006), Carvalho *et al.* (2006), Jefferis and Thupayagale (2008), Kasman *et al.* (2009), Sivakumar and Mohandas (2009), Bonga-Bonga and Makakabule (2010) and Maheshchandra (2012). The results from these studies are mixed with some finding long memory while others do not. Further, despite this large number of studies modelling the stock returns and volatility for the emerging markets, there are basically no forecasting studies in the long-memory framework on this series. The only known exceptions are studies by Kasman *et al.* (2009) and Sivakumar and Mohandas (2009). Kasman *et al.* (2009) investigate the presence of long memory in eight Central and Eastern European (CEE) countries' stock market, using the ARFIMA, GPH, FIGARCH and HYGARCH models. The results of these models indicate strong evidence of long memory both in conditional mean and in conditional variance. Moreover, the ARFIMA–FIGARCH model provides better out-of-sample forecast for the sampled stock markets. Sivakumar and Mohandas (2009) investigate the forecasting ability of ARFIMA–FIGARCH model using Indian stock returns and find that the ARFIMA–FIGARCH model performed better than the traditional Box and Jenkins ARIMA models.

Against this background, we contribute to this research area by first reinvestigating the existence or otherwise of long memory for the BRICS stock returns. Secondly, we contribute by predicting stock returns for the BRICS using a long-memory (ARFIMA) model. Thirdly, we compare the forecast ability of the ARFIMA with non-ARFIMA models. The autoregressive fractionally integrated moving average (ARFIMA) process is one of the best-known classes of long-memory models (Contreras-Reyes and Palma, 2013). The ARFIMA model possesses theoretical properties which lie between the two worlds of ARMA and integrated processes, and it is able to model both the short and long run dynamics of a time series (Sivakumar and Mohandas, 2009). Hence, the current study resorts to the use of ARFIMA models. The superiority of ARFIMA model in forecasting stock return for the BRICS is proved by comparing its performance with non-ARFIMA (AR, MA, ARMA and GARCH) models. We intend to contribute to the argument by Granger (1999) on the likelihood of I(d) processes falling into the 'empty box' category.[1]

The rest of the article is organized as follows. The next section describes the methodology. In Section III, the data, results and discussion are presented. Section IV concludes.

II. Methodology

In this section, we present the ARFIMA processes and the estimation and testing techniques used to investigate the predictability of stock returns.

ARFIMA: long-memory estimation

A typical autoregressive fractionally integrated moving average (ARFIMA) process is given as:

[1] By 'empty box', Granger means ARFIMA models have stochastic properties that essentially do not mimic the properties of the data (Bhardwaj and Swanson, 2006).

$$\gamma(L)(1-L)^d y_t = \Psi(L)\varepsilon_t \qquad (1)$$

where, L is the lag operator and the standard difference operator $(1-L)$ of an ARIMA process is replaced with a fractional difference operator $(1-L)^d$, where d denotes the degree of fractional integration or simply the fractional differencing parameter, ε_t is independently and identically distributed and the process is covariance stationary for $-0.5 < d < 0.5$, with mean reversion when $d < 1$. This model is a generalization of the fractional white noise process as described in Granger (1980), Granger and Joyeux (1980) and Hosking (1981), where, for the purpose of analysing the properties of the process, $\Psi(L)$ is set equal to unity.[2] Given that many time series exhibit gradually decaying autocorrelations, the merits of using ARFIMA models with hyperbolic autocorrelation decay patterns in financial time series modelling are many. The vital role of the hyperbolic decay property can be easily be illustrated by noting that

$$(1-L)^d = \sum_{j=0}^{\infty}(-1)^j \binom{d}{j}(L)^j = 1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + \ldots\ldots\ldots = \sum_{j=0}^{\infty} b_j(d) \qquad (2)$$

for any $d > -1$. For $d > 0$, the difference filter can also be developed further using a hypergeometric function as below:

$$(1-L)^d = \Lambda(-d)\sum_{j=0}^{\infty} L^k \Lambda(j-d)/\Lambda(j+1) = F(-\mathrm{d},1,1,\mathrm{L}),$$
$$\text{where } \mathrm{F(a,b,c,z)} = \Lambda(\mathrm{c})/[\Lambda(\mathrm{a})\Lambda(\mathrm{b})] \sum_{\mathrm{j}=0}^{\infty} \mathrm{z^j}\Lambda(a+j)\Lambda(b+j)/[\Lambda(c+j)\Lambda(j+1)]. \qquad (3)$$

It is should be noted that the plausible reason for the emergence of a various range of techniques for estimation and testing of long-memory models is due to the lack of a full proof of good method of estimation. Many of the tests used for long memory have been shown via finite sample experiments to perform quite poorly. Much of this evidence has been reported in the context of comparing one or two classes of estimators/tests, such as rescaled range (RR)-type estimators. In the current study, we employ a variety of estimators and tests. Specifically, we consider four widely used estimation methods and five different long-memory tests following Bhardwaj and Swanson (2006). The four long-memory estimators are the GPH, WHI, RR and AML estimators. Three of the estimation methods require first estimating d. Thereafter, an ARMA model is fitted to the filtered data by using maximum likelihood to estimate parameters and via the use of the Schwarz Information Criterion (SIC) for lag selection. Four of the five long-memory tests that we use when evaluating our time series are based on the GPH, RR, MRR and WHI tests. Note that of these, only the GPH and WHI tests are based directly upon examination of the d estimator, while the RR and MRR tests do not involve first estimating d. The fifth test that we use is the nonparametric short-memory test of Leybourne *et al.* (2003). Their test is based on the rate of decay of the autocovariance function. Specifically, the null hypothesis of the test is that the data are short memory (i.e. $\sum_{j=0}^{\infty}|\gamma_j| < \infty$; where γ_j is the autocovariance of y_t at lag j) and the test is based on the notion that one can distinguish between short- and long memory via knowledge of the rate at which $\gamma_j \to 0$ as $j \to \infty$.[3]

Predictive accuracy and testing

Most often, the ultimate goal of an empirical investigation is the specification of predictive models, and then a natural tool for testing for the presence of long memory is the predictive accuracy test. In this case, if an ARFIMA model can be shown to yield predictions that are superior to those from a variety of alternative linear (and nonlinear) models, then one has direct evidence of long memory, at least in the sense that the long-memory model is the best available 'approximation' to the true underlying DGP. Conversely, even if one finds evidence of long memory via application of the tests discussed above, then there is little use specifying long-memory models if they do not out predict simpler alternatives. There is a rich recent literature on predictive accuracy testing, most of which draws in one way or another on Granger and Newbold (1986), where simple tests comparing mean-square forecast errors (*MSFEs*) of pairs of alternative models under assumptions of normality are outlined. Perhaps the most important of the predictive accuracy tests that have been developed over the last 20 years is the Diebold and Mariano (DM, 1995) test. The DM statistic is:

$$\hat{d}_p = P^{-0.5}\frac{\sum_{t=R-h+1}^{T-1}\left(f(\hat{v}_{0,t+h}) - f(\hat{v}_{1,t+h})\right)}{\hat{\sigma}_p}, \qquad (4)$$

[2] See Baillie (1996) for a series of surveys on the properties of the ARFIMA process.
[3] See Bhardwaj and Swanson (2006) for technical details about the estimators and tests.

where R denotes the estimation period, P is the prediction period, f is a generic loss function, $h \geq 1$ is the estimate horizon, $\hat{v}_{0,t+h}$ and $\hat{v}_{1,t+h}$ are $h-$ step-ahead prediction errors for the models 0 and 1 (where model 0 is assumed to be the ARFIMA model), constructed using estimators.

The hypotheses of interest are the following:

$$H_0 : E(f(\hat{v}_{0,t+h}) - f(v_{1,t+h})) = 0$$

$$H_A : E(f(\hat{v}_{0,t+h}) - f(v_{1,t+h})) \neq 0 \quad (5)$$

The DM test, when constructed as outlined above for nonnested models, has a standard normal limiting distribution under the null hypothesis.[4] West (1996) shows that when the out-of-sample period grows at a rate not slower than the rate at which the estimation period grows (i.e. $\frac{P}{R} \to \pi$, with $0 < \pi \leq \infty$), parameter estimation error generally affects the limiting distribution of the DM test in stationary contexts. On the other hand, if $\pi = 0$, then the parameter estimation error has no effect. Additionally, Clark and McCracken (2001) point out the importance of addressing the issue of nestedness when applying DM and related tests. Although, the DM test does not have a normal limiting distribution under the null of noncausality when nested models are compared, the statistic can still be used as an important diagnostic in predictive accuracy analyses. Furthermore, the nonstandard limit distribution is approximated by a standard normal in many contexts (see McCracken, 2007 for tabulated critical values). In this regard, we use critical values obtained from the $\mathrm{N}(0, 1)$ distribution when carrying out DM tests.

Note that McCracken (2007) and Clark and McCracken (2001) assume stationary and correct specification under the null hypothesis, and that estimation is done using ordinary least squares. If we make the assumption of correct specification under the null, it implies that the ARFIMA and the non-ARFIMA models are the same. Hence $d = 0$; so that only the common ARMA components in the models remain, and therefore, the errors are short memory.

We also evaluate forecasts from ARFIMA and non-ARFIMA models using Clark and McCracken (2001) encompassing test which is designed for comparing nested models. The test statistic is given as:

$$ENC - t = (P-1)^{0.5} \frac{\bar{c}}{\left(P^{-1} \sum_{t=R}^{T-1} (c_{t+h} - \bar{c})\right)^{0.5}} \bar{c}, \quad (6)$$

where $c_{t+h} = \hat{v}_{0,t+h}(\hat{v}_{0,t+h} - \hat{v}_{1,t+h})$ and $\bar{c} = P^{-1} \sum_{t=R}^{T-1} c_{t+1}$. The test has the same hypotheses as the DM test, except that the alternative is $H_A : E(f(\hat{v}_{0,t+h}) - f(v_{k,t+h})) > 0$. If $\pi = 0$, the limiting distribution is $N(0, 1)$ for $h = 1$. The limiting distribution for h > 1 is nonstandard. However, as long as the Newey and West (1987)-type estimator (of the generic form given above for the DM test) is used when h > 1, then the tabulated critical values are quite close to the $N(0, 1)$ values, and hence, we use the standard normal distribution as a benchmark guide for all horizons.[5]

Predictive model selection

In this article, there are one-step-ahead, five-steps-ahead and 20-steps-ahead forecasts, when daily stock market data are examined, corresponding to 1-day-ahead, 1-week-ahead and 1-month-ahead predictions. Estimation is carried out as discussed above for ARFIMA models, and using maximum likelihood for non-ARFIMA models. More precisely, each sample of T observations is first split in half. The first-half of the sample is then used to produce 0:25T rolling (and recursive) predictions (the other 0:25T observations are used as the initial sample for model estimation) based on rolling (and recursively) estimated models (i.e. parameters are updated before each new prediction is constructed).

These predictions are then used to select a ‘best’ ARFIMA and a ‘best’ non-ARFIMA model, based on point out-of-sample mean-square forecast error comparison. At this juncture, the specifications of the ARFIMA and non-ARFIMA models to be used in later predictive evaluation are fixed. Parameters in the models may be updated, however. In particular, recursive and rolling *ex ante* predictions of the observations in the second half of the sample are then constructed, with parameters in the ARFIMA and non-ARFIMA ‘best’ models updated before each new forecast is constructed. Additionally, different models are constructed for each forecast horizon, as opposed to estimating a single model and iterating forward when constructing multiple-step-ahead forecasts. Reported DM and encompassing *t*-tests are thus based on the second-half of the sample and involve comparing only two models.

With regard to model selection, we use the SIC for choosing the best forecasting model as suggested by Inoue and Kilian (2006).

[4] We assume quadratic loss in our applications, so that $f(v_{0,t+h}) = v_{0,t+h}^2$, for example.
[5] See Clark and McCracken (2001) for an extended discussion.

III. Data and empirical evidence

The data have been sourced from the websites of the stock exchanges in each country considered. The data are the daily index representing a significant portion of the capitalization of each stock exchange. From this, daily stock returns were calculated. The data start in September 1995 and terminate over the period of 30 July 2012–6 September 2012 for the different countries. For Brazil, the Ibovespa is used and represents more than 80% of the number of trades and financial value traded, as well as representing over 70% of the total market capitalization of the stock exchange. Russia's All RTS index is used which comprises 50 preferred and common shares chosen according to capitalization. The Bombay sensitive index has been chosen in the case of India which tracks 30 stocks and is weighted according to market capitalization. China's Shanghai stock exchange A-share index is included; this index comprises stocks listed as A shares. Finally the FTSE/JSE All Share index is used to represent South Africa's stock returns; it comprises the top 99% of eligible listed companies and is weighted according to market capitalization.

The empirical estimation is based on the following models:

ARFIMA (p,d,q): $\gamma(L)(1-L)^d y_t = \beta + \Psi(L)\varepsilon_t$

where d takes fractional values.

Random Walk with a Drift: $y_t = \beta + y_{t-1} + \varepsilon_t$

$AR(p): \gamma(L)y_t = \beta + \varepsilon_t$

$MA(q): y_t = \beta + \Psi(L)\varepsilon_t$

$ARMA(p,q): \gamma(L)y_t = \beta + \Psi(L)\varepsilon_t$

ARIMA(p,d,q): $\gamma(L)(1-L)^d y_t = \beta + \Psi(L)\varepsilon_t$, where d can take integer values;

GARCH: $\gamma(L)y_t = \beta + \varepsilon_t$ where $\varepsilon_t = h_t^{0.5} v_t$ with $E(\varepsilon_t^2|\xi_{t-1}) = h_t = \varpi + \beta_1\varepsilon_{t-1}^2 + \ldots + \beta_q\varepsilon_{t-q}^2 + \beta_1 h_{t-1} + \ldots + \beta_p h_{t-p}$, and where ξ_{t-1} is the usual filtration of the data.

In these models, ε_t is the disturbance term $\gamma(L) = 1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p$, and $\Psi(L) = 1 - \theta_1 L - \theta_2 L^2 - \ldots - \theta_q L^q$, where L is the lag operator. All models (except ARFIMA models) are estimated using (quasi) maximum likelihood, with values of p and q chosen via use of the SIC, and integer values of d in ARIMA models selected via application of the augmented Dickey–Fuller t-test at a 5% level. Errors in the GARCH models are assumed to be normally distributed. ARFIMA models are estimated using the four estimation techniques listed above (GPH, RR, WHI and AML).

The results are presented in Table 1 with the number of observations used in each set of analysis included in brackets below the name of the relevant country. The analysis chooses the best ARFIMA and non-ARFIMA models given the data, and these are reported in the third and fifth columns of Table 1. The GPH and AML estimators were most often chosen for the ARFIMA models, whilst the AR-GARCH (1) and random walk models dominated the best non-ARFIMA models. The only exception in the case of the non-ARFIMA models was the 20-days-ahead rolling model for South Africa where an AR-GARCH (10) was chosen, thus having significantly more autoregressive terms than chosen for any other returns series. These results are *ex ante* estimates as described earlier. The size of d for Russia, India and China is frequently greater than 0.5, especially at horizons of 5- and 20-days ahead irrespective of the estimation scheme (i.e. rolling or recursive). This falls largely in line with results presented in Table 2 of Bhardwaj and Swanson (2006). The standard errors obtained from the reestimation of d for each forecast and reported in parenthesis in column four are relatively small. Additionally, all the forecasts for South Africa and Brazil fall below 0.5, and the rolling estimates are particularly low at less than 0.1, implying the presence of covariance stationary in this process.

The results of the DM test, reported in the sixth column of Table 1 give the results of a test that compares the MSFE of the best ARFIMA model with the best non-ARFIMA model. Negative values of the DM statistics indicate that the point MSFE associated with the ARFIMA model is lower than that for the non-Africa model since the former is taken as model zero. The results for every country are negative, which offhand suggest that the MSFE for the ARFIMA models are consistently lower than the non-ARFIMA models, adding further support to the use of ARFIMA models for stock market return forecasting. Most of the test statistics are significant at all conventional levels.[6] These are the DM test statistics for India's 5-days-ahead recursive and rolling forecasts and South Africa's 20-days-ahead rolling forecast. Thus, other than the exceptional cases identified, according to the DM test the ARFIMA models are preferred. In terms of the forecast encompassing (ENC-t) test, which tests as to whether the non-ARFIMA model is nested within the ARFIMA model, the findings suggest that there is little reason to reject the null hypothesis of nestedness in most cases. There are two exceptions where the null is not accepted (suggesting that the non-ARFIMA is the more accurate forecasting model) and that is for the 5-days-ahead rolling models for Brazil and the 20-days-ahead

[6] The normal distribution, N(0,1), has been used as a rough guide for significance in the case of the DM and ENC-t statistics.

Table 1. Analysis of absolute returns for Brazil, Russia, India, China and South Africa

		ARFIMA model	*d*	SE	Non-ARFIMA model	MSFE ratio	DM	ENC-*t*
Brazil (4173)	1-day-ahead recursive	AML(1,1)	0.3770	0.0199	GARCH(1,1)	0.0192	–2.4281**	–1.5147
	5-days-ahead recursive	AML(1,1)	0.3770	0.0199	RW	0.5692	–7.4267***	0.7984
	20-days-ahead recursive	AML(1,1)	0.3770	0.0199	GARCH(1,1)	0.0138	–1.9397*	–3.6880
	1-day-ahead rolling	AML(1,1)	–0.0053	0.2259	GARCH(1,1)	0.0198	–2.4687**	0.1836
	5-days-ahead rolling	AML(1,1)	–0.0053	0.2259	RW	0.5726	–7.4583***	1.2997*
	20-days-ahead rolling	AML(1,1)	–0.0053	0.2259	GARCH(1,1)	0.0144	–1.9793**	–0.4706
Russia (4240)	1-day-ahead recursive	AML(2,1)	0.1501	0.0203	GARCH(1,1)	0.0289	–2.9199***	–1.8090
	5-days-ahead recursive	GPH(1,1)	0.6629	0.0464	GARCH(1,1)	0.0194	–2.3574**	–0.9369
	20-days-ahead recursive	GPH(1,1)	0.6629	0.0464	GARCH(1,1)	0.0215	–2.4350**	–1.5285
	1-day-ahead rolling	GPH(1,1)	0.7364	0.0897	GARCH(1,1)	0.0300	–2.9600***	0.2044
	5-days-ahead rolling	GPH(1,1)	0.7364	0.0897	GARCH(1,1)	0.0206	–2.4194**	0.1602
	20-days-ahead rolling	WHI(1,1)	0.6205	0.0192	GARCH(1,1)	0.0224	–2.4824**	–0.9490
India (4214)	1-day-ahead recursive	GPH(1,1)	0.5383	0.0392	RW	0.5456	–7.0939***	–1.7616
	5-days-ahead recursive	GPH(1,1)	0.5383	0.0392	GARCH(1,1)	0.0194	–1.5157	–1.6006
	20-days-ahead recursive	GPH(1,1)	0.5383	0.0392	RW	0.5666	–5.6930***	0.3456
	1-day-ahead rolling	GPH(1,1)	0.6371	0.0620	GARCH(1,1)	0.0258	–1.9965**	–1.0815
	5-days-ahead rolling	GPH(1,1)	0.6371	0.0620	GARCH(1,1)	0.0175	–1.6148	–1.8913
	20-days-ahead rolling	GPH(1,1)	0.6371	0.0620	RW	0.5701	–5.6747***	–0.0101
China (4119)	1-day-ahead recursive	AML(0,3)	0.2866	0.0217	GARCH(1,1)	0.0190	–2.5017**	–0.1928
	5-days-ahead recursive	WHI(2,1)	0.5333	0.0068	RW	0.5125	–7.8821***	–1.6206
	20-days-ahead recursive	GPH(2,1)	0.5862	0.0468	RW	0.5458	–8.0691***	0.8741
	1-day-ahead rolling	AML(0,3)	0.3661	0.0217	GARCH(1,1)	0.0235	–2.4255**	–1.1795
	5-days-ahead rolling	GPH(2,1)	0.6288	0.0068	GARCH(1,1)	0.0155	–1.9475*	–0.7882
	20-days-ahead rolling	WHI(2,1)	0.5655	0.0468	RW	0.5474	–8.0705***	0.6950
South Africa (4250)	1-day-ahead recursive	AML(4,1)	0.2586	0.0563	RW	0.5119	–7.0051***	–3.5658
	5-days-ahead recursive	AML(4,1)	0.2586	0.0563	RW	0.5712	–9.4633***	1.0349
	20-days-ahead recursive	AML(4,1)	0.2586	0.0563	RW	0.5650	–6.6615***	1.5153*
	1-day-ahead rolling	AML(4,1)	0.0363	0.1399	RW	0.5067	–6.9875***	–1.5163
	5-days-ahead rolling	AML(4,1)	0.0363	0.1399	RW	0.5692	–9.4958***	1.0624
	20-days-ahead rolling	AML(4,1)	0.0363	0.1399	GARCH(1,1)	0.0004	–1.4076	–1.3305

Notes: ***, ** and * represent significance of the test statistics at the 1%, 5% and 10% levels, respectively. These are based on the standard normal distribution. In the case of the DM statistic, a two-tailed test is conducted such that the critical values are 2.54, 1.96 and 1.65, respectively. The ENC-*t* test is a one-tailed test with critical values 2.33, 1.65 and 1.29, respectively. The MSFE ratio calculates the ratio between the MSFE for the ARFIMA model relative to the non-ARFIMA model.

models recursive models for South Africa. Thus, the results point to a great extent to the existence of a long-memory process in the absolute daily returns of the stock markets considered and that an ARFIMA model has a role to play in forecasting exercises for stock returns at the 1-day-ahead, 5-days-ahead and 20-days-ahead horizons.

The final column compares the forecast errors of the ARFIMA and non-ARFIMA models. Thus, any figure greater than one suggests that the MSFE for the ARFIMA models is higher than for the non-ARFIMA models and *vice versa*. The results suggest that for all models at all horizons for all countries, that the best ARFIMA model produces a lower MSFE than the best non-ARFIMA model. This lends further support to the evidence that ARFIMA models are better predictors than the non-ARFIMA options.

IV. Conclusion

This article investigates the existence of long-memory processes in the absolute returns of indices for the Brazilian, Russian, Indian, Chinese and South African stock markets. In order to verify whether the true data generating process is better represented by ARFIMA or non-ARFIMA models, the study further compared the forecasts generated by these set of models. The best ARFIMA and best non-ARFIMA were first selected using part of the data. The remaining part of the data was used to produce *ex ante* forecast from the best selected models using both recursive and rolling estimation schemes. We also employ a variety of estimators and forecast evaluation tests.

Our results provide strong evidence supporting the existence of long memory in daily stock returns for the BRICS countries over different horizons. This is inconsistent with the weak-form market efficiency, implying that the BRICS stock index consists of the impact of news and shocks occurred in the recent past. Hence, speculative earnings could be gained via predicting stock prices. This findings contrasts the argument of Granger (1999) on the likelihood of I(d) processes falling into the 'empty box' category but consistent with those of Bhardwaj and Swanson (2006). The evidence also suggests that the ARFIMA models are better at forecasting daily stock market returns than the non-ARFIMA models. These findings hold true across all the BRICS countries whose economies differ in size, nature and sophistication, based on a number of tests. Thus, the usefulness of ARFIMA models, specifically for the purposes of forecasting at a number of horizons is further supported with the evidence presented in this article. These findings would be helpful to the investors, financial managers and regulators dealing with the BRICS stock markets. Understanding the sources of long memory in the stock market could also assist the regulators in improving its efficiency.

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