On the Convergence of an Inexact Primal-Dual Interior Point Method for Linear Programming

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Abstract. The inexact primal-dual interior point method which is discussed in this paper chooses a new iterate along an approximation to the Newton direction. The method is the Kojima, Megiddo, and Mizuno globally convergent infeasible interior point algorithm. The inexact variation is shown to have the same convergence properties accepting a residue in both the primal and dual Newton step equation also for feasible iterates.

1 Introduction

For the standard primal-dual linear programming problem the optimality conditions are the Karush-Kuhn-Tucker (KKT) conditions:

$$F(x, y, z) \equiv \begin{pmatrix} Ax - b \\ A^T y + z - c \\ XZe \end{pmatrix} = 0, \quad x \geq 0,\ z \geq 0, \tag{1}$$

where A is an m-by-n matrix of full rank m, b an m-vector, c an n-vector,z an n-vector, $X = \text{diag}(x)$, $Z = \text{diag}(z)$ and e is the vector of all ones in $\Re^n$.

Bellavia [1] proved global convergence of an inexact interior point method. Ito, Kelly, and Sachs [7] and Ito [6] discuss an inexact primal-dual interior point iteration for linear programs in functional spaces. Mizuno and Jarre [9] proved global and polynomial-time convergence of an infeasible interior point algorithm using inexact computation. Portugal *et al.* [11] presented a truncated primal-infeasible dual-feasible interior point algorithm for linear programming. Portugal *et al.* [10] presented a truncated primal-infeasible dual-feasible interior point algorithm for solving monotone linear complementarity problems. The methods suggested in [7, 10, 11] have a major drawback of remaining primal-feasible once they become primal-feasible.

Throughout this paper we use the following notation: For any vector x, x^k denotes x at the k-th (interior point) iteration. Similary for any matrix X or real number η, X^k and η^k denotes X and η at the k-th iteration. We have adopted the notation $(u, v, w) = (u^T, v^T, w^T)^T$, so for any vectors $x \in \Re^n, y \in \Re^m, (x, y) \in \Re^{m+n}$.

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2 Inexact Computation

The perturbed KKT conditions for (1) with a positive μ is the nonlinear system

$$F_\mu(x,y,z) \equiv \begin{pmatrix} Ax-b \\ A^T y + z - c \\ XZe - \mu e \end{pmatrix} = 0, \quad x \geq 0,\ z \geq 0. \tag{2}$$

The parameter μ is referred to as the μ-complementarity parameter. Newton's method defines the equation of directional change (the Newton step equation)

$$F'_{\mu^k}(x^k,y^k,z^k)\begin{pmatrix} \Delta x^k \\ \Delta y^k \\ \Delta z^k \end{pmatrix} = -F_{\mu^k}(x^k,y^k,z^k). \tag{3}$$

If the linear system of equation is solved approximately, then equation (3) has a residual error r^k given by

$$F'_{\mu^k}(x^k,y^k,z^k)\begin{pmatrix} \Delta x^k \\ \Delta y^k \\ \Delta z^k \end{pmatrix} = -F_{\mu^k}(x^k,y^k,z^k) + r^k. \tag{4}$$

The residual will be partioned into the primal infeasibility $\bar{r}^k$, dual infeasibility $\hat{r}^k$, and deviation in complementarity $\tilde{r}^k$, $r^k = (\bar{r}^k, \hat{r}^k, \tilde{r}^k)$. Since $F' = F'_\mu$ and $F_\mu = F - \mu^k\,(0,0,e)$ an inexact Newton step of (4) is an approximate solution of the Newton step equation derived from (1)

$$F'(x^k,y^k,z^k)\begin{pmatrix} \Delta x^k \\ \Delta y^k \\ \Delta z^k \end{pmatrix} = -F(x^k,y^k,z^k) + r_g^k, \text{ where } r_g^k = \mu^k \begin{pmatrix} 0 \\ 0 \\ e \end{pmatrix} + r^k. \tag{5}$$

For the centering parameter β_1, typical choice of the μ-complementarity parameter at iteration k is $\mu_k = \beta_1 \frac{(x^k)^T z^k}{n}$, where $\beta_1 \in [0,1]$. Bellavia [1] observed that $\|F(x^k,y^k,z^k)\|_2 \geq \frac{(x^k)^T z^k}{\sqrt{n}}$. If $\|r^k\|_2 \leq \eta^k (x^k)^T z^k$ then

$$\|r_g^k\|_2 \leq \mu^k\sqrt{n} + \|r^k\|_2 \leq \frac{(x^k)^T z^k}{\sqrt{n}}\left(\beta_1 + \eta^k\sqrt{n}\right) \leq \left(\beta_1 + \eta^k\sqrt{n}\right)\|F(x^k,y^k,z^k)\|_2.$$

Hence the sequence $\{\beta_1 + \eta^k\sqrt{n}\}$ can be regarded as a forcing sequence of inexact Newton methods [3] applied to the nonlinear system (1). Let

$$\|r^k\|_2 \leq \eta^k (x^k)^T z^k \ \text{ for } \ \eta^k < (1-\beta_1)/\sqrt{n}. \tag{6}$$

Since (6) is overly restrictive, we will later show that solving the linear system (4) with an accuracy $\|r^k\|_1 \leq \eta^k\,(x^k)^T z^k$ for $0 \leq \eta^k < 1 - \beta_1$ will be sufficient to achieve convergence. Bellavia [1] establishes global convergence results for an inexact interior point method by interpreting it as an inexact Newton